Linear Channel Diffusion for Image Denoising

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Robust smoothing is important in many applications for low level image processing. Common approaches to reduce noise in images are diffusion-based methods like isotropic non-linear diffusion [2], anisotropic diffusion [4] or field of experts [3]. Another robust smoothing approach can be formulated using channel representations [1]. This method consists of three steps, an encoding step where the image is decomposed into channels, a smoothing step within the channels followed by a reconstruction.

In this work our contribution is a novel variational formulation which combines a diffusion scheme with the channel framework. Therefore, we define the energy

$$E(u) = \frac{1}{2}\int_\Omega (u(x,y) - u_0(x,y))^2 \; dxdy + \lambda \frac{1}{2}\int_\Omega |\nabla u(x,y)|^2 \; dxdy \qquad (1)$$

where λ is a regularization parameter, Ω denotes the image domain and u_0 is the observed image. The one-to-one transformation between the channel representation and the image domain is done by a linear combination of N channel centers c and suitable basis functions B:

$$u(x,y) = \sum_{i=1}^{N} c_i B(u(x,y) - c_i) \qquad (2)$$

where i denote the channel index. Here we consider quadratic B-splines as basis functions.

Figure 1: Qualitative denoising results of different denoising methods.

To find the stationary point of (1) we compute the variational derivative using Neumann boundary conditions and obtain the corresponding Euler-Lagrange equation. The solution to the functional $E(u)$, expressed in its channel representation, is then the solution of the initial-boundary value problem for the PDE:

$$\left\{ \begin{array}{rcl} \partial_t u - c^t \left[\frac{1}{2} \left(B'' B'^t + B' B''^t \right) |\nabla u|^2 + \lambda \left(B' B'^t \right) \Delta u \right] c & = & 0 \\ u(x, y, 0) & = & u_0 \\ \langle \nabla u, n \rangle & = & 0 \end{array} \right. \tag{3}$$

Equation (3) is solved using a forward Euler scheme where the derivatives are approximated using finite differences.

Qualitatively the proposed channel diffusion framework compares favorably to other diffusion methods. In figure 1 an image was corrupted with additive Gaussian noise and salt and pepper noise. For moderate noise levels the proposed method outperforms non-linear diffusion and channel smoothing in terms of retaining image details better.

References

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