

Linear Channel Diffusion for Image Denoising

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Robust smoothing is important in many applications for low level image processing. Common approaches to reduce noise in images are diffusion-based methods like isotropic non-linear diffusion [2], anisotropic diffusion [4] or field of experts [3]. Another robust smoothing approach can be formulated using channel representations [1]. This method consists of three steps, an encoding step where the image is decomposed into channels, a smoothing step within the channels followed by a reconstruction.

In this work our contribution is a novel variational formulation which combines a diffusion scheme with the channel framework. Therefore, we define the energy

$$E(u) = \frac{1}{2} \int_{\Omega} (u(x, y) - u_0(x, y))^2 dx dy + \lambda \frac{1}{2} \int_{\Omega} |\nabla u(x, y)|^2 dx dy \quad (1)$$

where λ is a regularization parameter, Ω denotes the image domain and u_0 is the observed image. The one-to-one transformation between the channel representation and the image domain is done by a linear combination of N channel centers c and suitable basis functions B :

$$u(x, y) = \sum_{i=1}^N c_i B(u(x, y) - c_i) \quad (2)$$

where i denote the channel index. Here we consider quadratic B -splines as basis functions.

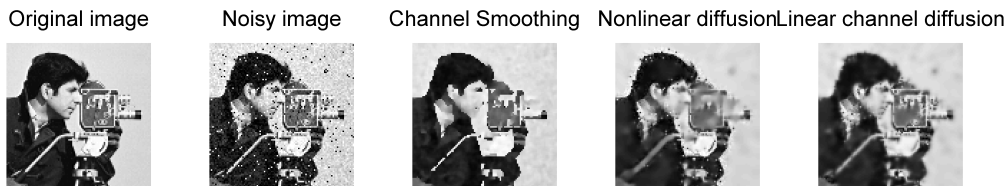


Figure 1: Qualitative denoising results of different denoising methods.

To find the stationary point of (1) we compute the variational derivative using Neumann boundary conditions and obtain the corresponding Euler-Lagrange equation. The solution to the functional $E(u)$, expressed in its channel representation, is then the solution of the initial-boundary value problem for the PDE:

$$\begin{cases} \partial_t u - c^t \left[\frac{1}{2} (B'' B'^t + B' B''^t) |\nabla u|^2 + \lambda (B' B'^t) \Delta u \right] c = 0 \\ u(x, y, 0) = u_0 \\ \langle \nabla u, n \rangle = 0 \end{cases} \quad (3)$$

Equation (3) is solved using a forward Euler scheme where the derivatives are approximated using finite differences.

Qualitatively the proposed channel diffusion framework compares favorably to other diffusion methods. In figure 1 an image was corrupted with additive Gaussian noise and salt and pepper noise. For moderate noise levels the proposed method outperforms non-linear diffusion and channel smoothing in terms of retaining image details better.

References

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