The crystallographic Radon transform and the pole figure inversion

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In this talk we consider a special type of Radon transform that appears in crystallography and in material sciences. A well known problem in material sciences is the determination of material properties. This method is also known as texture analysis. Texture analysis is the analysis of the statistical distribution of orientations of crystals within a specimen of a polycrystalline material, which could be metals or rocks.

A crystallographic orientation is a set of crystals symmetrically equivalent rotations between an individual crystal and the specimen. The orientation distribution by volume requires a measure of the volume of a polycrystalline specimen carrying crystal gains with orientations within a volume element $\Delta G \subset G$ of the subgroup G of all feasible $G \in SO(3)$. Assuming that the measure possesses a probability density function $f: G \to R_+$, then

$prob(g \in \Delta G) = \int_{\Delta G} f(g)\, dg$ and f is referred to as the orientation density function (ODF).

In X-ray defraction experiments, the orientation density function f cannot by directly measured but with a texture goniometer the pole density function (PDF) $P(h,r)$ can be sampled. The PDF represents that a fixed crystal direction h or its antipodal –h statistically coincides with the specimen direction r. The PDF is the tomographic projection of an ODF given by

$Pf(h,r) = \frac{1}{2}\big(Rf(h,r) + Rf(-h,r)\big)$, where the crystallographic Radon transform is given by

$$(Rf)(h,r) = \frac{1}{2\pi} \int_{\{g \in SO(3): h = gr\}} f(g) dg$$

Now, the determination of the ODF from pole figures is an inverse problem. We determine the range and image of the crystallographic Radon transform, explain some inversion formulae which give different insides into the problem. We will demonstrate connection between the Radon transform and wavelets and finally we describe an exact reconstruction formula for bandlimited functions, which uses only a finite number of samples of their Radon transform.