On solving integral equations of the first kind arising in interior source methods for electromagnetic scattering problems

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We discuss solving three-dimensional electromagnetic scattering problems by interior source methods, where one looks for solutions as integrals over an auxiliary surface inside the scattering body.

Let Ω be a compact set in $\mathbf{R}^3$ with boundary Γ. Consider the following problem

$$\begin{aligned}
&\operatorname{curl}\operatorname{curl} E - k^2 E = 0 && \text{in } \mathbf{R}^3 \setminus \Omega,\\
&E \times n = f \times n && \text{on } \Gamma,\\
&\frac{x}{|x|} \times \operatorname{curl} E(x) + ikE(x) = o\left(\frac{1}{|x|}\right) && \text{as } |x| \to \infty,
\end{aligned} \tag{1}$$

where E is an unknown scattered field and f is a given vector field. This problem describes the scattering of an electromagnetic field from a perfectly conducting body Ω. To solve the equations, we choose a closed surface γ inside Ω and look for solutions of the form

$$E(x) = \int_\gamma \nabla_y \frac{e^{ik|x-y|}}{|x-y|} \times u(y)\, dS_y, \quad x \in \mathbf{R}^3 \setminus \gamma,$$

where u is a tangential vector field on γ. Any E of this form satisfies the differential equations and the radiation condition. To satisfy the boundary conditions, u must be the solution of the following integral equation:

$$\int_\gamma \nabla_y \frac{e^{ik|x-y|}}{|x-y|} \times u(y)\, dS_y \times n(x) = f(x) \times n(x), \quad x \in \Gamma. \tag{2}$$

Results about existence and uniqueness of the solution of the integral equation are presented. If the boundary is not analytic, then the equation (2) is not solvable in ordinary function spaces, but we can still prove the density of the range of the integral operator.

Since the solution of (1) is of interest, we discretize the integral equation using linear combinations of Dirac's δ-functions with supports on γ. To calculate the coefficients of the δ-functions, various methods can be used, e.g. collocation or least squares methods.

If the boundary of the scatterer is analytic, then for specific choices of the auxiliary surface and meshes the convergence is exponential in the number of variables. For two-dimensional scattering problems the exponential convergence of the collocation method with δ-functions is proved in [1]. For three-dimensional problems convergence has been proved only for the least squares methods [2].

Some computations have also been performed, in both analytic and nonanalytic cases.

References

[1] U. Kangro, Convergence of Collocation Method with Delta Functions for Integral Equations of First Kind, *Integr. Equ. Oper. Theory*, **66**, 265–282 (2010).

[2] U. Kangro, Solution of three-dimensional electromagnetic scattering problems by interior source methods, *AIP Conf. Proc.*, **1479**, 2328–2331 (2012).