Reconstruction of wave speed in a seismic inverse problem

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We analyze the inverse problem, originally formulated by Dix in geophysics, of reconstructing the wave speed inside a domain from boundary measurements associated with the single scattering of seismic waves.
We consider a domain M with a varying and possibly anisotropic wave speed which we model as a Riemannian metric g. For our data, we assume that M contains a dense set of point scatterers and that in a subset $U\subset M$, modeling the domain that contains the measurement devices, e.g, on the Earth's surface is seismic measurements, we can produce sources and measure the wave fronts of the single scattered waves diffracted from the point scatterers. The inverse problem we study is to recover the metric g in M up to a change of coordinates. To do this we show that the shape operators related to wave fronts
produced by the point scatterers within M satisfy a certain system of differential equations which may be solved along geodesics of the metric. In this way, assuming we know g as well as the shape operator of the wave fronts in the region U, we may recover g in certain coordinate systems (i.e. Riemannian normal coordinates centered at point scatterers). This generalizes the well-known geophysical method of Dix to metrics which may depend on all spatial variables and be anisotropic. In particular, the novelty of this solution lies in the fact that it can be used to reconstruct the metric also in the presence of the caustics.

The results have been done in collaboration with Maarten de Hoop, Sean Holman, Einar Iversen, and Bjorn Ursin.