B-spline slice-by-slice solution of 3*D*-vector tomography problem.

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In this talk the 3D-vector tomography problem is considered in the following formulation. Let a bounded domain in three-dimensional space be filled by a medium without refraction (probing rays permeate along straight lines). In the domain some vector field *v(x)* is given. It is require to find this field by its known ray transform. In calculating the ray transform along the ray only projection of the desired field on the ray makes contribution to the result. Therefore problem of reconstruction of vector fields *v(x)* by its ray transform *[Iv]* has no unique solution. Namely, the operator *[I]* possesses not trivial kernel consisting of potential vector fields with vanishing potentials on the boundary of domain. Therefore only the solenoidal part of the field *v(x)* can be recovered from *[Iv]*.

The problem is overdetermined in dimension, because we try to recover vector field *v(x),* where $x \in R^3$, from function *[Iv]* on the four-dimensional manifold of oriented lines. Therefore it is natural to pose the problem of recovering *v(x)* from incomplete data $[Iv]|_{M^3}$, where M^3 is some three-dimensional submanifold of manifold of oriented lines. In this talk approximation of solenoidal part of vector field is built by known ray transform, calculated along the lines parallel to one of the coordinate planes. Two different coordinate planes are sufficient for the uniqueness (2P-problem), but three coordinate planes are needed for the stable reconstruction of a solenoidal part of vector field (3P-problem).

In this talk we propose algorithm of solving of the problem of recovery of a solenoidal part of vector field, which is given in unit ball. The algorithm of solving is based on the least squares method where we use a finite basis consisting of B-splines as basis function. In that sense the method can be seen as a projection method for minimizing an L_2-data fitting term.

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