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Background
VAR(p) under Student-...
Cook's Local influence...
Influence analysis for...
An empirical study
Conclusion and Future...

Influence diagnostics in a vector autoregressive model

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Outline

- Background
- VAR(p) model under Student-t distribution
- Cook's local influence method
- Influence analysis for VAR(p)
- An empirical study
- Future work

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1 Background

As is well known, VAR(p) model is

$$y_t = v + A_1 y_{t-1} + A_2 y_{t-2} + ... + A_p y_{t-p} + u_t,$$

in which

- ★ $y_t = (y_{1t}, ..., y_{kt})^T$ are presample vectors;
- ★ $v = (v_1, ..., v_k)^T$ is intercept vector;
- ★ A_i are coefficient matrices;
- ★ u_t are independent error vectors.

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VAR(p) model

We use the matrix notations to represent the VAR(p) model:

$$Y = XB + U,$$

in which

- $Y = \left(y_T, y_{T-1}, \cdots, y_{p+1}\right)^T$;
- $B = \left(v, A_1, A_2, \cdots, A_p\right)^T$;
- X is a $(T-p) \times (1+kp)$ matrix.

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Comments for VAR(p)

VAR(p) modeling is an essential technique to describe multiple time series, and is applied in many disciplines such as economics and finance.

See e.g.

Sims (1980), Tiao and Box (1981), Lütkepohl (2005) and Tsay (2010).

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Statistical Diagnostic

Statistical diagnostic analysis for regression and time series models is equally important in data analysis and applications.

See e.g.

Cook (1986), Atkinson et al. (2004), Kleiber and Zeileis (2008) and Liu and Welsh (2011).

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Why to do statistical diagnostic ?

- Influence diagnostics is the study of how relevant minor perturbations affect model fit and inference results.
- To identify those outliers can help us to obtain a suitable model.
- Influence diagnostics has become a useful methodology for statistical analysis.

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Pioneering work

Cook, R.D., 1986. Assessment of local influence (with discussion). Journal of the Royal Statistical Society, Ser. B 48, 133–169.

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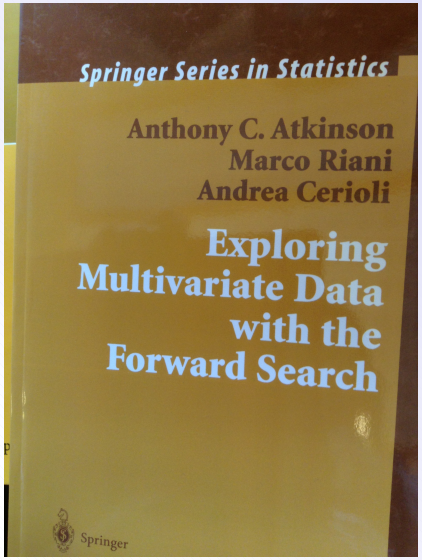
Atkinson's book



Atkinson, A.C., Riani, M., Cerioli, A., 2004. Exploring multivariate data with the forward search. Springer, Berlin.

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Subsequently

- Billor and Loynes (1993), Wu and Luo (1993) and Poon and Poon (1999) suggested alternative approaches for assessing local influence.
- Shuangzhe Liu, Ejaz Ahmed and L.Y.Ma (2009) investigated influence diagnostics in the linear regressive model with stochastic constraints.

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Applications to time series

- Tsai and Wu (1992) and Kim and Huggins (1998) investigated the local influence of linear regression models with first-order autoregressive or heteroskedastic error structures.
- Schall and Dunne (1991), Lu et al. (2012) and Zevallos et al. (2012) considered influence diagnostics for time series models.
- Shuanzghe Liu (2004), Shuangzhe Liu and Heyde (2008) and Liu and Neudecker (2009) researched influence diagnostics for time series models.
- Influence diagnostics for GARCH processes were discussed by Zhang and King (2005), Dark, Zhang and Qu (2010).

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Our recent work

- Last year, Yonghui Liu, Guocheng Ji and Shuangzhe Liu (2014) considered influence diagnostics for a VAR(p) model under normal distribution.
- We established the normal curvature and slope diagnostics for the VAR(p) and use the Monte Carlo method to obtain benchmark.
- We also gave an empirical study using the VAR model. The data come from the monthly simple returns of IBM and S&P500 index.

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Student-t distribution !

- It is well-known that stock returns and other data feature with heavier tails than the normal distribution. In order to cope with such data involving heavy- or light-tailed characteristics, Student-t and some other elliptical distributions may be adopted.
- Paula et al. (2009) for diagnostic analysis for linear models with first-order autoregressive elliptical errors, and Russo et al. (2012) for influence diagnostics in heteroskedastic or autoregressive nonlinear elliptical models.
- Diagnostic checks for ARCH time series models were studied by Shuangzhe Liu and co-authors, and Kwan et al. (2012).

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Our work

In this talk, we will establish influence diagnostics for a VAR(p) model under Student-t distribution and give an empirical study to illustrate the effectiveness of the proposed diagnostics.

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2 VAR(p) under Student-t distribution

VAR(p) model is as follows:

$$Y = XB + U.$$

$U = (u_T, u_{T-1}, ..., u_{p+1})^T$ is a $(T-p) \times k$ matrix. We suppose U follows a $(T-p) \times k$ Student-t distribution. i.e.

$$U \sim t_{(T-p)k}(0, \Sigma, \gamma),$$

then u_t follows a k-dimensional multiple Student-t distribution of mean zero, covariance matrix $\frac{\gamma}{\gamma-2}\Sigma$ and γ degrees of freedom.

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Probability density of t-distribution

The probability density function of U is given by the following equation:

$$f(U) = C(1 + \frac{tr(U\Sigma^{-1}U^T)}{\gamma})^{-\frac{(T-p)K+\gamma}{2}},$$

where C is a constant,

$$C = \frac{\Gamma\frac{(T-p)K+\gamma}{2}}{\Gamma(\frac{\gamma}{2})(\pi\gamma)^{\frac{(T-p)K}{2}}|\Sigma|^{\frac{T-p}{2}}}.$$

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Log-likelihood function of VAR(p)

- Let $\theta = (b^T, s^T)^T$ denote the vector of parameters, in which $b = vec(B)$ and $s = vech(\Sigma)$, then b and s are $k + k^2p$ and $\frac{(k+1)k}{2}$ vector.

- The relevant part of the log-likelihood function $L(\theta)$ of VAR(p) can be expressed as follows:

$$L(\theta) = -\frac{T-p}{2}\ln|\Sigma| - \frac{(T-p)k+\gamma}{2}\ln(1+\frac{x}{\gamma}),$$

where

$$x = tr\left[(Y - XB)\Sigma^{-1}(Y - XB)^T\right] = tr\left(U\Sigma^{-1}U^T\right)$$

.

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Derivatives of $L(\theta)$

Theorem 1: The derivatives of $L(\theta)$ with respect to b and to Σ can be expressed as

$$\frac{\partial L(\theta)}{\partial b} = \frac{(T-p)K+\gamma}{\gamma}(1+\frac{x}{\gamma})^{-1}(\Sigma^{-1}\otimes X^T)vec(U),$$

$$\frac{\partial L(\theta)}{\partial \Sigma} = -\frac{T-p}{2}\left[2\Sigma^{-1}-diag(\Sigma^{-1})\right]$$

$$+\frac{(T-p)K+\gamma}{2\gamma}(1+\frac{x}{\gamma})^{-1}\left[2\Sigma^{-1}U^TU\Sigma^{-1}-diag(\Sigma^{-1}U^TU\Sigma^{-1})\right],$$

where $x = tr\left[(Y-XB)\Sigma^{-1}(Y-XB)^T\right]$.

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Maximum Likelihood estimations

From Theorem 1, we can obtain the ML estimators of B and Σ as

$$\widehat{B} = (X^T X)^{-1} X^T Y,$$

$$\widehat{\Sigma} = \frac{1}{T-p} \widehat{U}^T \widehat{U},$$

where $\widehat{U} = Y - X\widehat{B}$.

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3 Cook's Local influence method

- Let $\omega = (\omega_1, ..., \omega_q)^T$ denote a $q \times 1$ vector of perturbations confined to some open subset of R^q and ω_0 denote a no-perturbation vector.
- $L(\theta)$ and $L(\theta|\omega)$ present the log-likelihood functions of the postulated (i.e. unperturbed) and the perturbed models, respectively. Note that $L(\theta) = L(\theta|\omega_0)$.

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Cook displacement

The idea of the local influence method is to investigate how much the inference is affected by those minor changes in the corresponding perturbations. In Cook (1986), his likelihood displacement used to assess the influence of the perturbation ω is defined as

$$LD(\omega) = 2\left[L(\widehat{\theta}) - L(\widehat{\theta}_\omega)\right].$$

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T+1 dimensional space

- We introduce a perturbation as follows $\omega = \omega_0 + \alpha l$, where ω_0 is the point of no-perturbation, and α measures the magnitude of the perturbation in the direction l.
- The influence of the perturbation vector on the likelihood displacement can be examined on a graph in $T + 1$ dimensional space spanned by ω and $LD(\omega)$ and expressed as $\alpha(\omega) = (\omega_1, \omega_2, ..., \omega_T, LD(\omega))$.

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Information matrix and Δ matrix

By Cook (1986), the local influence of the perturbation vector at $\omega = \omega_0$ can be examined by the normal curvature, which is

$$C_l = 2|l^T \ddot{F} l| = 2|l^T (\Delta^T \ddot{L}^{-1} \Delta) l|,$$

where l is a $T \times 1$ vector of unit length,

$$\ddot{F} = \frac{\partial^2 L(\theta|\omega)}{\partial\omega\partial\omega^T}, \quad \Delta = \frac{\partial^2 L(\theta|\omega)}{\partial\theta\partial\omega^T}, \quad \ddot{L} = \frac{\partial^2 L(\theta|\omega)}{\partial\theta\partial\theta^T},$$

and $-\ddot{L}$ is the information matrix for the postulated model.

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Cook's method

Cook's method is to make the local influence analysis by finding maximum curvature C_{max} and $\Delta^T \ddot{L}^{-1} \Delta$, with the largest absolute eigenvalue λ_{max} and its associated eigenvector l_{max}. If the absolute value of the i^{th} element of l_{max} is the largest, then the i^{th} observation in the data may be most influential.

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4 Influence analysis for VAR(p)

4.1. Information matrix

Theorem 2 For VAR(p) model with Student-t distribution, the information matrix is

$$\ddot{L} = \begin{pmatrix} \ddot{L}_{11} & 0 \\ 0 & \ddot{L}_{22} \end{pmatrix},$$

where

$$\ddot{L}_{11} = -\left(\widehat{\Sigma}^{-1} \otimes X^T X\right),$$

$$\ddot{L}_{22} = -\frac{T-p}{2} D_K^T (\widehat{\Sigma}^{-1} \otimes \widehat{\Sigma}^{-1}) D_K - \frac{(T-p)^2}{2((T-p)K+\gamma)} D_K^T vec(\Sigma^{-1}) vec(\Sigma^{-1})^T D_K$$

and D_K is the $K^2 \times \frac{K(K+1)}{2}$ duplication matrix.

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4.2. Perturbation of Case-weights

Theorem 3 For VAR(p) model under the perturbation of case-weights, we have

$$\Delta = \frac{\partial^2 L(\theta,\omega)}{\partial\theta\partial\omega^T}|_{\theta=\widehat{\theta},\omega=\omega_0} = \begin{pmatrix} 2\left[\left(\widehat{\Sigma}^{-1}\widehat{U}^T\right)\otimes X^T\right]S \\ \Delta_2 \end{pmatrix},$$

where S is an $(T-p)^2\times(T-P)$ selection matrix with $d_\omega vecW = Sd\omega$, and

$$\Delta_2 = -\frac{T-p}{(T-p)K+\gamma}D_K^T vec(\widehat{\Sigma}^{-1})\left[vec\left(\widehat{U}\widehat{\Sigma}^{-1}\widehat{U}^T\right)\right]^T S + D_K^T\left[\left(\widehat{\Sigma}^{-1}\widehat{U}^T\right)\otimes\left(\widehat{\Sigma}^{-1}\widehat{U}^T\right)\right]S.$$

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4.3. Perturbation of positive defined matrix Σ

Theorem 4 For VAR(p) model under the perturbation of Σ, we have

$$\Delta = \frac{\partial^2 L(\theta,\omega)}{\partial\theta\partial\omega^T}|_{\theta=\widehat{\theta},\omega=\omega_0} = \begin{pmatrix} \left[\left(\widehat{\Sigma}^{-1}\widehat{U}^T\right)\otimes X^T\right] S \\ \Delta_2 \end{pmatrix},$$

where S is an $(T-p)^2 \times (T-P)$ selection matrix with $d_\omega vecW = Sd\omega$, and

$$\Delta_2 = -\frac{T-p}{2\left[(T-p)K+\gamma\right]} D_K^T vec(\widehat{\Sigma}^{-1}) \left[vec\left(\widehat{U}\widehat{\Sigma}^{-1}\widehat{U}^T\right)\right]^T S + \frac{1}{2} D_K^T \left[\left(\widehat{\Sigma}^{-1}\widehat{U}^T\right)\otimes\left(\widehat{\Sigma}^{-1}\widehat{U}^T\right)\right] S.$$

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4.4. Perturbation of Data

If the time series are stable, then there is a vector μ such that

$$\mu = E(y_t) = (I_K - A_1 - A_2 - \cdots - A_p)^{-1} v,$$

in this case, VAR(p) model can be rewritten as follows

$$y_t - \mu = A_1 (y_{t-1} - \mu) + A_2 (y_{t-2} - \mu) + ... + A_p (y_{t-p} - \mu) + u_t,$$

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For convenience, we regard $y_t - \mu$ as y_t, then we obtain a no-intercept VAR(p) model as follows

$$y_t = A_1 y_{t-1} + A_2 y_{t-2} + ... + A_p y_{t-p} + u_t.$$

Next, we will introduce a perturbation on data.

1. $\omega = (\omega_T, \omega_{T-1}, \cdots, \omega_1)$ is perturbation vector,
2. $\omega_0 = (0, 0, \cdots, 0)$ is no-perturbation vector.

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VAR(p) model under perturbation of data becomes

$$\begin{bmatrix} y_{1,t} \\ y_{2,t} \\ \vdots \\ y_{K,t} \end{bmatrix} + \begin{bmatrix} \omega_t \\ \omega_t \\ \vdots \\ \omega_t \end{bmatrix} = A_1 \left(\begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ \vdots \\ y_{K,t-1} \end{bmatrix} + \begin{bmatrix} \omega_{t-1} \\ \omega_{t-1} \\ \vdots \\ \omega_{t-1} \end{bmatrix} \right) + \cdots$$

$$+ A_p \left(\begin{bmatrix} y_{1,t-p} \\ y_{2,t-p} \\ \vdots \\ y_{K,t-p} \end{bmatrix} + \begin{bmatrix} \omega_{t-p} \\ \omega_{t-p} \\ \vdots \\ \omega_{t-p} \end{bmatrix} \right) + \begin{bmatrix} u_{1,t} \\ u_{2,t} \\ \vdots \\ u_{K,t} \end{bmatrix}.$$

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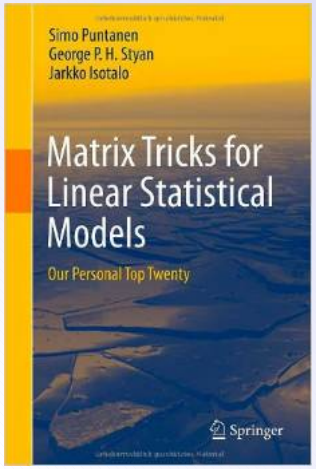
Simo's book



Simo Puntanen, George P.H. Styan, and Jarkko Isotalo. Matrix Tricks for Linear Statistical Models: Our Personal Top Twenty. Springer 2011.

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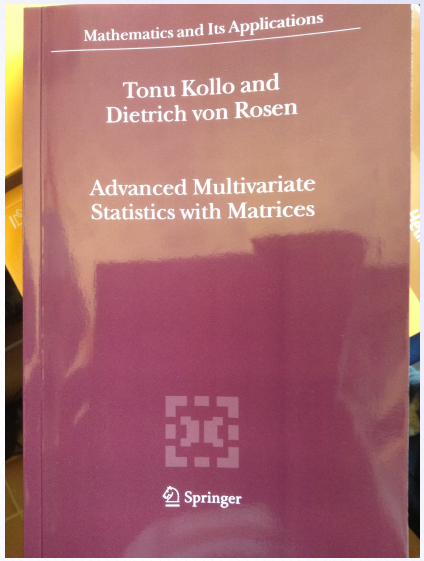
Dietrich's book



Tonu Kollo, Dietrich Von Rosen. Advanced Multivariate Statistics with Matrices. Springer 2005.

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Kaitai Fang's book

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Log-likelihood function

By using the matrix tricks, the relevant part of the pertured log-likelihood function becomes

$$L(\theta,\omega) = -\frac{(T-p)K+\gamma}{2}\ln\left[1+\frac{\sum_{t=p+1}^{T}\left(\mu_t(\omega)\right)^T\Sigma^{-1}\mu_t(\omega)}{\gamma}\right],$$

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Matrix tricks

where

$$u_t(\omega) = (y_t + G_t\omega) - B^T(x_t + FK_t\omega) = y_t - B^T x_t + \left(G_t - B^T FK_t\right)\omega,$$

in which

$$F = I_p \otimes e, \quad G_t = (0, \cdots, 0, e, 0, \cdots, 0)_{p\times T}, \quad e = (1, 1, \cdots, 1)^T,$$

$$B = B_{kp\times k} = (A_1, A_2, \cdots, A_p)^T, \quad K_t = (K_t)_{p\times T} = (e_{t-1}, e_{t-2}, \cdots, e_{t-p})^T,$$

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Matrix tricks

e is a $k \times 1$ column vector, and is located in the $T - t + 1$ column of G_t, and

$$e_t = (0, \cdots, 0, 1, 0, \cdots, 0)^T$$

is a $T \times 1$ column vector, and 1 is located in the $T - t + 1$ row of e_t.

We denote

$$R_t = G_t - B^T F K_t.$$

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Theorem 5 For VAR(p) model under the perturbation of data, we have

$$\Delta = \frac{\partial^2 L(\theta,\omega)}{\partial\theta\partial\omega^T}|_{\theta=\widehat{\theta},\omega=\omega_0} = \begin{pmatrix} \Delta_1 \\ \Delta_2 \end{pmatrix},$$

where

$$\Delta_1 = \sum_{t=p+1}^{T} \Phi^{-1}\left[x_t \otimes \left(\widehat{\Sigma}^{-1}\widehat{R}_t\right)\right] + \sum_{t=p+1}^{T}\left[\left(\widehat{\Sigma}^{-1}\widehat{u}_t\right)\otimes(FK_t)\right],$$

$$\Delta_2 = -\frac{T-p}{(T-p)K+\gamma} D_K^T vec(\widehat{\Sigma}^{-1}) \sum_{t=p+1}^{T}\left[vec\left(\widehat{R}_t^T\widehat{\Sigma}^{-1}\widehat{u}_t\right)\right]^T + D_K^T \sum_{t=p+1}^{T}\left[\left(\widehat{\Sigma}^{-1}\widehat{u}_t\right)\otimes\left(\widehat{\Sigma}^{-1}\widehat{R}_t\right)\right],$$

in which D_k is a duplication matrix and Φ is a commutation matrix such that $vec(B) = \Phi^T vec(B^T)$.

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5 An empirical study

We choose weekly log-return data

- Standard & Poor's 500 Index (SPX)
- Chevron shares (CVX)

from January 12, 2007 to August 1, 2014

as two-dimensional vectors to construct VAR model, and then do local influence analysis.

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Why to choose SPX and CVX ?

- The correlation between CVX and SPX is 0.6892.
- CVX account for a large proportion of the market value of the SPX, the volatility of CVX would affect the trend of SPX to a certain extent.
- CVX is in the energy industry which rise and fall is heavily depend on the macroeconomic situation, so SPX's trend would have a significant impact on CVX.

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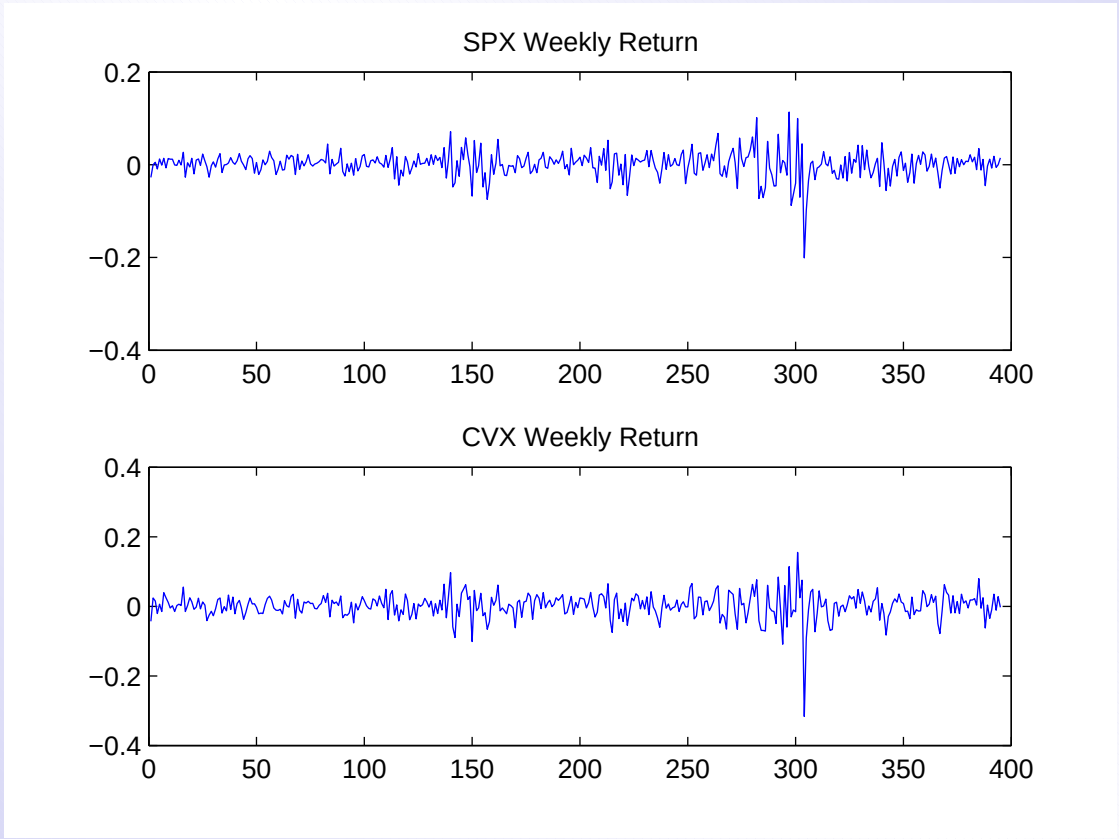
SPX and CVX's weekly log-return



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How to get VAR model's order p ?

- **Step 1.** Assuming the two dimension data subject to VAR(p) model $(p = 1, 2, ...)$.
- **Step 2.** We test the VAR(i) model versus a VAR(i-1) model. The test statistic is

$$M(i) = -(T - i - 3.5)\ln(\frac{|\widehat{\Sigma}_i|}{|\widehat{\Sigma}_{i-1}|})$$

where

$$\widehat{\Sigma}_i = \frac{1}{T - 2i - 1}\sum_{t=i+1}^{T} \widehat{u}_t^{(i)}(\widehat{u}_t^{(i)})^T$$

$$\widehat{u}_t^{(i)} = y_t - \widehat{v} - \widehat{A}_1^{(i)} y_{t-1} - \widehat{A}_2^{(i)} y_{t-2} - \ldots - \widehat{A}_i^{(i)} y_{t-i}$$

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Order $p = 3$!

For our model, $M(i)$ is asymptotically distributed as a $\chi^2(k^2)$- distribution with $k^2 = 4$ is degree of freedom.

After calculation, $M(3) = 22.42$, $M(4) = 2.28$ and $M(5) = 2.20$, 99% quantile of the $\chi^2(4)$ is 13.27, so judge the order of VAR model is 3.

This method is called partial autocorrelation function method (PACF).

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Parameters estimation

The estimate of coefficients B and Σ of the VAR(3) model are as follows:

$$\widehat{B} = \begin{pmatrix} 0.0011 & -0.1351 & 0.0659 & 0.2696 & -0.1879 & 0.1275 & -0.2028 \\ 0.0018 & -0.1520 & 0.1206 & 0.3018 & -0.2223 & -0.0229 & -0.1981 \end{pmatrix}$$

$$\widehat{\Sigma} = \begin{pmatrix} 7.4076 & 7.4970 \\ 7.4970 & 12.4188 \end{pmatrix} \times 10^{-4}$$

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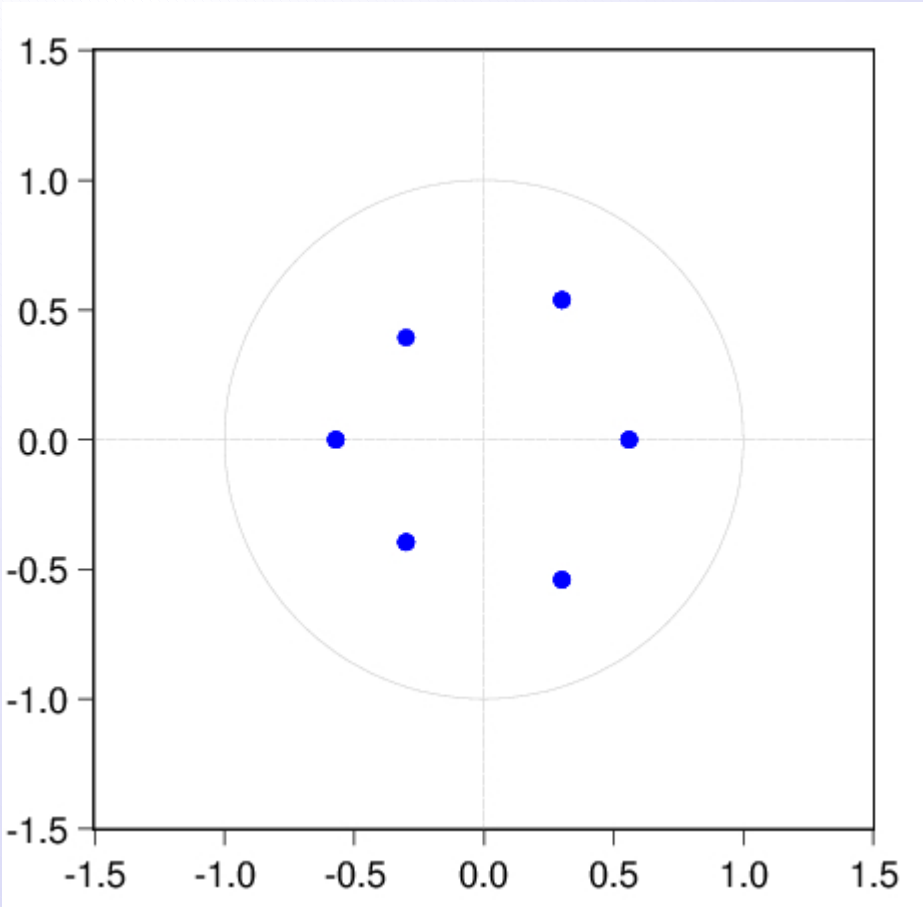
Unit root test for VAR(3)



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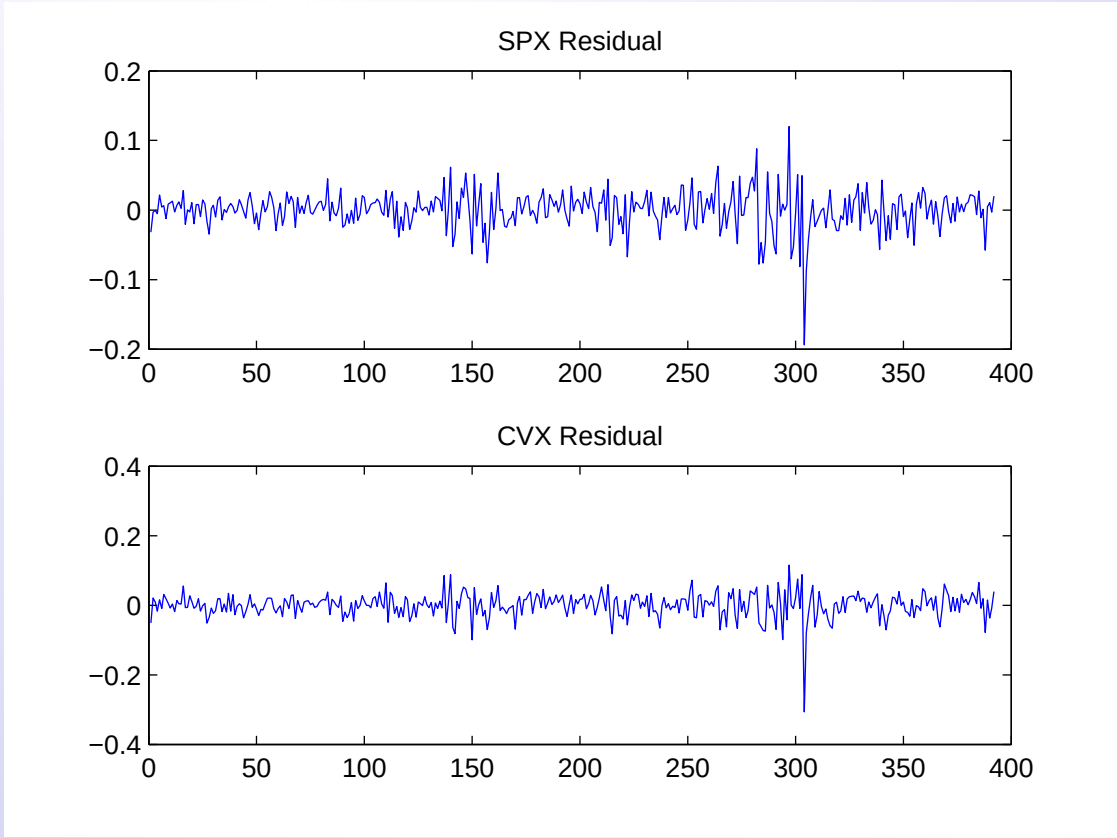
SPX and CVX's error of VAR(3) model



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How to get the DF of errors ?

Next, we determine the degree of freedom (DF) of the two-dimensional error vectors which assumed follow bivariate Student-t distribution. It's difficult to do when put them together, but we can use one-dimensional Student-t distribution to fit them separately to get the degree of freedom of VAR(3) model.

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Adjust variance before comparison

- **Step 1** Standardize the errors and adjust it's variance to $\frac{\gamma}{\gamma-2}$;
- **Step 2** Adjust standard normal distribution's variance similarly;
- **Step 3** Put the the non-parametric estimate of the density function of adjusted-standard errors, adjusted standard normal distribution and standard t distribution in one figure in order to facilitate comparison.

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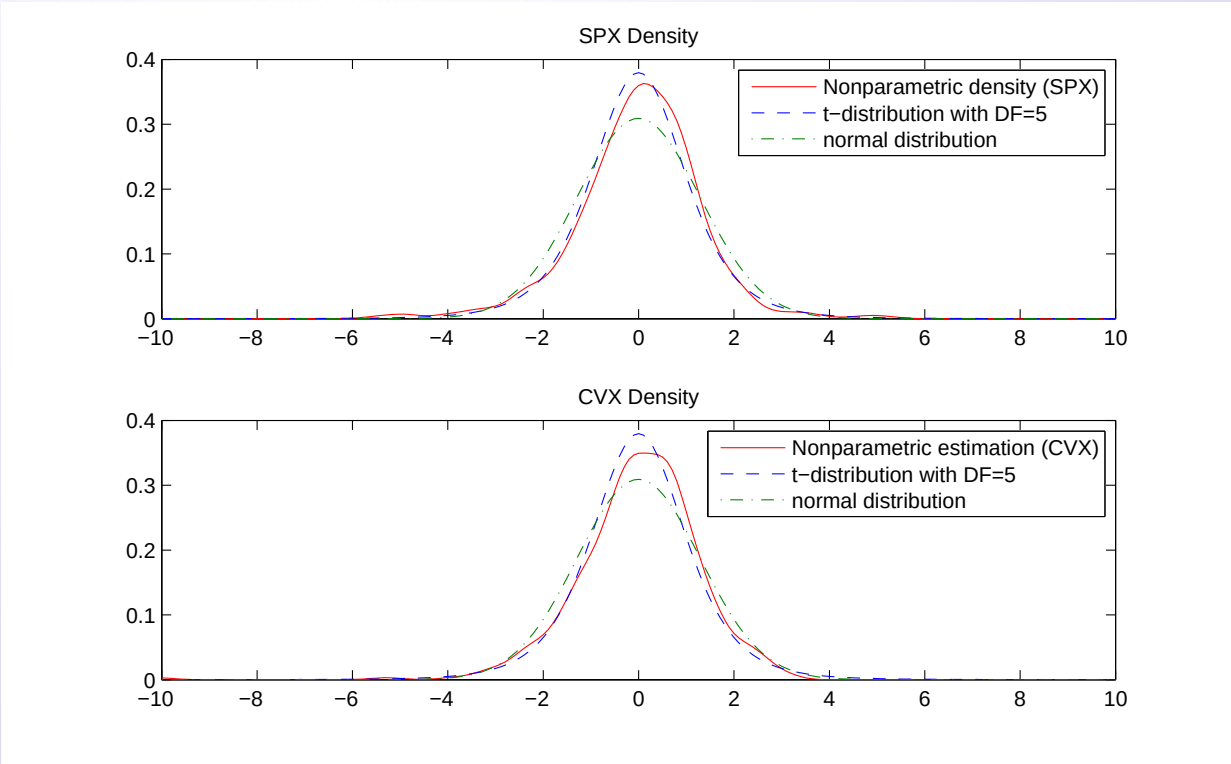
SPX and CVX error's density function



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DF=5 !

we can clearly see that the errors distribution are far from normal distribution and used the t distribution with $DF = 5$ to fit them are appropriate. So we can say that the DF of their joint distribution is 5, at least it is suitable.

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Build benchmark through Monte Carlo

In our simulations we repeatedly get 300 groups of sample of size 395 based on the VAR(3) model, with parameter values chosen as the estimates obtained by fitting this model to the weekly log-returns of SPX and CVX. Note that the simulation results depend on the parameter values, the lag orders and DF of error.

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Monte Carlo simulation processes

- **Step 1.** We calculate SPX and CVX bivariate VAR(3) model's maximum curvatures under three perturbation schemes.

perturbation scheme	Case-weights	Positive defined matrix Σ	data
curvature	35.5107	8.8777	13851.6438

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Monte Carlo simulation processes

- **Step 2.** Based on the $\widehat{B}$ and $\widehat{\Sigma}$, we generate 300 groups of samples which each of them have the same length with SPX and CVX data $(T = 395)$, their errors follow the bivariate t distribution with poistive defined matrix equals $\widehat{\Sigma}$ and $DF = 5$. Then we calculate each group of sample's maximum curvatures under three perturbation scheme.
- **Step 3.** Setting a corridor equals 10%. we only retain the sample's curvature that within the range of corresponding curvature product $(1 \pm 10\%)$, because samples which meet the conditions
  - not only follow same model with SPX and CVX
  - but also samples' data quality are similar to them

  Then we calculate their eigenvectors corresponding to their maximum eigenvalue, namely the diagnostic vector l_{max}, and sort each l_{max}'s elements in descending, we record the 95% quantile element.

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Monte Carlo simulation processes

- **Step 4.** We compute the mean of the 95% quantile elements we got in **Step 3** and take it as our benchmark for corresponding SPX and CVX's diagnostic vector under each perturbation schemes.

The table below present the samples' curvature which meet conditions in **Step 4** and show the corresponding 95% quantile element of their diagnostic vectors.

Note that first and second perturbation schemes, case weights and positive defined matrix Σ, must present similar result because they have multiple relationships, so we just present the result of perturbation of case weights and perturbation of data.

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Monte Carlo result: case-weights

curvature	95% quantile	curvature	95% quantile	curvature	95% quantile
33.2885	0.0383	36.3459	0.0400	33.3735	0.0370
34.0789	0.0403	38.9673	0.0266	32.7423	0.0339

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Monte Carlo result: data

curvature	95% quantile	curvature	95% quantile	curvature	95% quantile
14250.8099	0.0980	14390.1401	0.0909	14890.3229	0.0966
13644.3660	0.0952	15003.5618	0.0958	13632.9200	0.1010
12678.7665	0.0938	12957.9381	0.0986	12576.3955	0.1044
15009.7747	0.0955	13496.3293	0.1003	14303.5212	0.0973
15216.0843	0.0992	12716.5882	0.0918	14575.8197	0.0918
13518.1310	0.0979	13340.3909	0.0964	13683.1048	0.0964
13546.9280	0.0995	12816.9854	0.0895	13418.6248	0.0878
13260.8195	0.0980	12868.8172	0.0996	13192.0402	0.0961
13503.1438	0.0948	14603.7853	0.0897	13058.4816	0.0972
13551.7631	0.0964	14104.4126	0.1025	14596.4929	0.1025
13438.5104	0.1003	13959.6706	0.0945	14409.0097	0.0916
14222.6832	0.0956	14217.5935	0.0926	13047.9183	0.0926
13357.7475	0.0952	12628.8742	0.0969	14870.7724	0.0973
12595.0022	0.1015	13132.4288	0.1016	13406.8479	0.0917
14194.7706	0.0979	15070.4622	0.0965	12950.7900	0.0946
12513.3655	0.0979	14272.9599	0.0968	14409.2156	0.0913
13848.0453	0.0957	13289.0821	0.0908	12615.5390	0.0950
13631.5415	0.0994	14420.5859	0.0964	12852.2761	0.0910
13806.2920	0.0927	14509.1242	0.0961	14333.7112	0.1015
14598.5307	0.0983	13254.5708	0.1022	13728.0566	0.0980
14396.1561	0.1018	15050.9737	0.0957	13769.1830	0.0992
14650.1019	0.0939	13728.3195	0.0902	13715.9265	0.0967
12672.2857	0.0933	13413.6701	0.0927	13922.9108	0.0882
13960.0008	0.0931	13526.0139	0.0954	13253.1751	0.0903
12910.3036	0.0958	12514.1064	0.1000	14767.4575	0.0953

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Benchmark

To get our benchmark for diagnostic vector of the SPX and CVX's bivariate VAR(3) model, we select mean of those samples' 95% quantile that shown in table 3 and 4, The table below summarises the maximum, minimum, mean and standard deviation of these 95% quantile elements

	case-weights	data
mean	0.0360	0.0960
std	0.0052	0.0037
max	0.0403	0.1044
min	0.0266	0.0878

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Local influence analysis for SPX and CVX

The red line represents the benchmark value which determines whether the observation is significantly influential. The observation is significantly influential when its diagnostic value is beyond the red lines.

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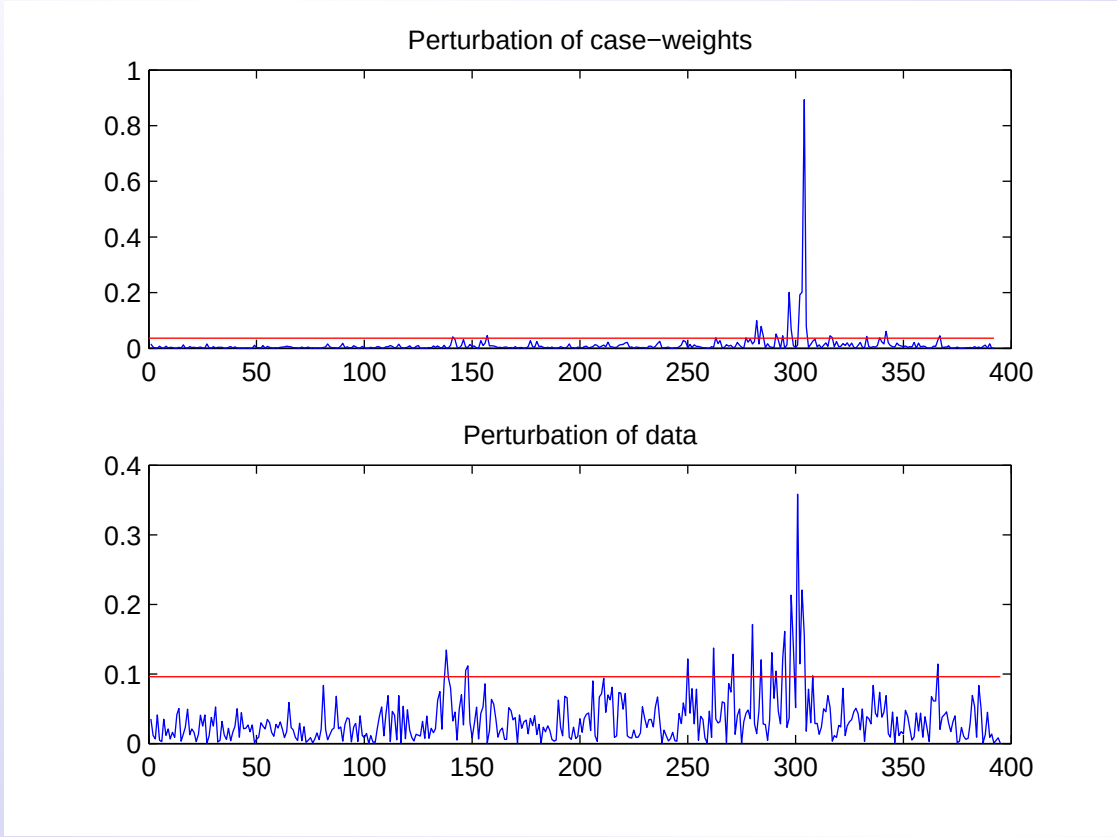
Diagnostics vectors of perturbations of case weights and data



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Summary of the Curvature-Based Diagnostics

No.	Time	SPX's return	CVX's return	Case-weights	data
138	2011-12-16	-2.87%	-3.31%		*
141	2011-11-25	-4.80%	-5.88%	*	
147	2011-10-14	5.81%	6.23%		*
148	2011-10-07	2.10%	1.94%		*
157	2011-08-05	-7.46%	-6.64%	*	
250	2009-10-23	-0.75%	-0.17%		*
262	2009-07-31	0.84%	1.51%		*
263	2009-07-24	4.05%	4.96%	*	
271	2009-05-29	3.56%	3.40%		*
277	2009-04-17	1.51%	-4.76%	*	
280	2009-03-27	5.98%	6.27%		*
282	2009-03-13	10.17%	7.66%	*	
284	2009-02-27	-4.65%	-6.94%	*	*
289	2009-01-23	-2.16%	-1.29%		*
291	2009-01-09	-4.55%	-4.96%	*	*
294	2008-12-19	0.92%	-10.89%	*	*
295	2008-12-12	0.42%	5.97%		*
297	2008-11-28	11.36%	11.41%	*	
298	2008-11-21	-8.76%	-3.06%	*	*
299	2008-11-14	-6.40%	-1.07%		*
301	2008-10-31	9.98%	15.47%		*
302	2008-10-24	-7.02%	2.47%	*	*
303	2008-10-17	4.49%	7.53%	*	*
304	2008-10-10	-20.08%	-31.67%	*	*
305	2008-10-03	-9.85%	-9.11%	*	
308	2008-09-12	0.75%	4.89%		*
316	2008-07-18	1.70%	-6.96%	*	
317	2008-07-11	-1.87%	-6.69%	*	
333	2008-03-20	3.16%	-2.53%	*	
339	2008-02-08	-4.70%	-3.99%	*	
342	2008-01-18	-5.56%	-8.29%	*	
366	2007-08-03	-1.79%	-5.03%		*
367	2007-07-27	-5.02%	-7.81%	*	

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Historical events

1. Many of these points are concentrated in before and after the 2008 financial crisis, such as on September 7, 2008, the U.S. Treasury Department announced to take over Fannie MAE and Freddie MAC. On October 3, 2008, the bush administration signed a total of up to 700 billion dollars financial rescue plan.

2. We even diagnosed with that data in August 3, 2007 is a strong influence points, the one thing which happened during that week – the 10th largest mortgage service providers (Residential mortgage investment company) in the United States filed for bankruptcy protection – exposed the risk of subprime mortgage bonds, it become the early warning signal for the arrival of the 2008 financial crisis.

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Model modification

We substitute two-dimensional zero vectors for the SPX and CVX return data diagnosed from above table, so we get a new 2-dimensional time series, Rebuild a VAR model for the new data, we get the modified model's lag order equals 8. We only present error of the modified model.

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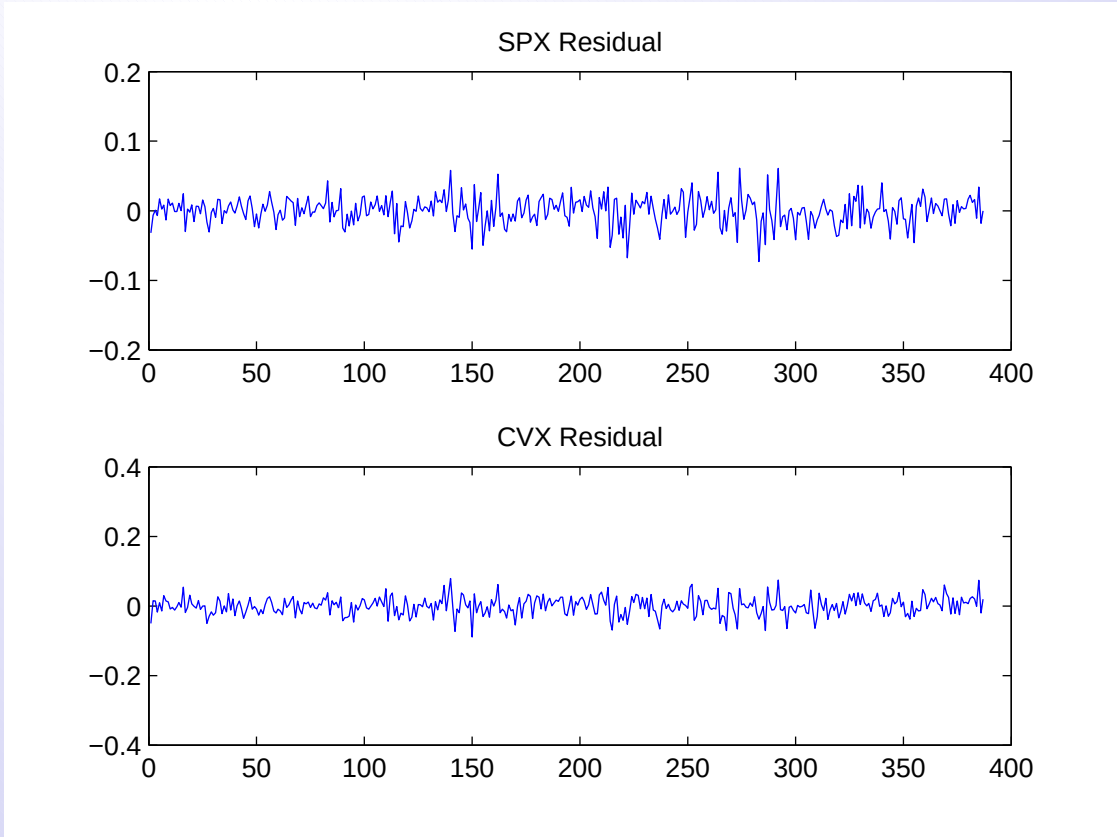
Errors of modified model



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6 Conclusion and Future work

- We have established the normal curvature diagnostics for the VAR model under three perturbation schemes.
- An empirical study applying the VAR model have been done to show that the effectiveness of the proposed diagnostics.

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Problems

- From model modification, we see that the new VAR(p) model is order 8, it is too big. Maybe ARMA model is a suitable model. Further, do diagnostics for VARMA or VARCH and V-GARCH are some good topics.

- Given multivariate data follow a joint Student t-distribution, how to determine and test the joint distribution's degree of freedom ?

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Thanks for your attention !

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