Geometry beyond limits

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Abstract: In my lecture I report on various limit problems involving the p-Laplace and related operators as $p \to \infty$ and $p \to 1$. The limit problems lead to interesting geometrical questions. One of the results says that the first (and positive) eigenvalue λ_p and eigenfunction u_p of

$$\Delta_p u + \lambda_p |u_p|^{p-2} u_p = 0 \quad \text{in } \Omega, \qquad u = 0 \quad \text{on } \partial\Omega,$$

converge to the Cheeger constant $\lambda_1 = h(\Omega)$ and a function u_1 amost all of whose level sets $\{u_1 > t\}$ are Cheeger sets of Ω, i.e. they minimize the ratio of perimeter over volume among all subsets of Ω. Cheeger sets are interesting geometrical objects. These and related results were obtained in part with my coauthors M.Belloni, V. Ferone, V.Fridman, P.Juutinen and T. Lachand-Robert.

Another result (with H.Shahgholian) deals with minimizers u_p of the functional

$$E_p(v) = \int_{\mathbb{R}^n} \frac{1}{p} |\nabla v|^p + \frac{p-1}{p} \chi_{\{v>0\}} \; dx$$

(among $v \in W^{1,p}(\mathbb{R}^n)$ and with $v \equiv 1$ on K) as $p \to 1$ or $p \to \infty$. They are p-harmonic in their support (minus K) and satisfy $u_p = 0$ as well as Bernoulli's condition $|\nabla u_p| = 1$ on the free boundary of their support. For $p = 1$ this free boundary minimizes surface area among all sets that contain K, and for $p \to \infty$ the support minimizes volume among all sets containing a 1-neighbourhood of K.

Finally I present a family of operators A_p with the properties that $A_p = \frac{1}{p} A_1 + \frac{p-1}{p} A_\infty$ is a convex combination of A_1 and A_∞ and that $u_t - A_p u = 0$ includes physically relevant equations such as the heat equation and the equation for mean curvature flow $u_t - |\nabla u| \operatorname{div}(\nabla u / |\nabla u|) = 0$. This may be yet another way to say caloric.