

Positive solutions to quasilinear elliptic equations with Hardy-type potentials in exterior domains

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Abstract

We study the existence and nonexistence of positive (super)solutions to the singular quasilinear p -Laplace equation

$$-\Delta_p u - \frac{\mu}{|x|^p} u^{p-1} = \frac{u^q}{|x|^s}$$

in exterior domains of \mathbf{R}^N ($N \geq 2$). Here $p \in (1, +\infty)$ and $\mu \leq C_H$, where C_H is the Hardy constant. We provide a sharp characterization of the set of $(q, s) \in \mathbf{R}^2$ such that the equation has no positive (super) solutions.

The proofs are based on the explicit construction of appropriate barriers and involve the analysis of asymptotic behavior of superharmonic functions associated to the p -Laplace operator with Hardy type potentials, comparison principles and an improved version of Hardy's inequality in exterior domains.