Antonio F. Costa

On the connectedness of the branch locus and p-gonal locus in the moduli space and its compactification

Abstract Consider the moduli space $\mathcal{M}_g$ of Riemann surfaces of genus $g \geq 2$ and its Deligne-Munford compactification $\overline{\mathcal{M}_g}$. We are interested in the branch locus $\mathcal{B}_g$ for $g > 2$, i.e., the subset of $\mathcal{M}_g$ consisting of surfaces with automorphisms. It is well-known that the set of hyperelliptic surfaces (the hyperelliptic locus) is connected in $\mathcal{M}_g$ but the set of (cyclic) trigonal surfaces is not. By contrast, we show that for $g \geq 5$ the set of (cyclic) trigonal surfaces is connected in $\overline{\mathcal{M}_g}$. To do so we exhibit an explicit nodal surface that lies in the completion of every equisymmetric set of 3-gonal Riemann surfaces. For $p > 3$ the connectivity of the p-gonal loci becomes more involved. We show that for $p \geq 11$ prime and genus $g = p - 1$ there are one-dimensional strata of cyclic p-gonal surfaces that are completely isolated in the completion $\overline{\mathcal{B}_g}$ of the branch locus in $\overline{\mathcal{M}_g}$.
Results in collaboration with G. Bartolini, M. Izquierdo, H. Parlier, A. M. Porto