An inverse fractional abstract Cauchy problem with nonlocal conditions

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Abstract

This note is devoted to the study of an inverse Cauchy problem in a Hilbert space $H$ for the abstract fractional differential equation of the form:

$$\frac{d^\alpha u(t)}{dt^\alpha} = A u(t) + f(t) g(t),$$

with the nonlocal initial condition:

$$u(0) = u_0 + \sum_{k=1}^{p} c_k u(t_k),$$

and the overdetermination condition:

$$(u(t), v) = w(t),$$

where $(.,.)$ is the inner product in $H$, $f$ is a real unknown function $w$ is a given real function, $u_0$, $v$ are given elements in $H$, $g$ is a given abstract function with values in $H$, $0 < \alpha \leq 1$, $u$ is unknown, and $A$ is a linear closed operator defined on a dense subset of $H$.

It is supposed that $A$ generates a bounded semigroup. An application is given to study an inverse problem in a suitable Sobolev space for general fractional parabolic partial differential equations with unknown source functions.

Keywords and phrases: Fractional abstract differential equations, nonlocal initial conditions, inverse Cauchy problem.

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