Three-spheres theorems

Tomasz Adamowicz, *Polish Academy of Sciences, Warsaw, Poland*

We will discuss the classical convexity result for planar subharmonic functions due to Hadamard and its generalizations to the setting of elliptic PDEs and systems of PDEs in Euclidean domains and in the Carnot groups of Heisenberg type. The talk is partially based on a joint work with Ben Warhurst.

Elliptic and parabolic boundary Harnack principles for nonsmooth domains

Hiroaki Aikawa, *Hokkaido University, Sapporo, Japan*

The (elliptic) boundary Harnack principle (BHP) is a comparison between two positive harmonic functions vanishing on a portion of the boundary. The validity of the BHP heavily depends on the geometry of the domain. For a smooth domain the BHP readily holds by the comparison with the distance function. For a Lipschitz domain such a comparison does not hold. Nevertheless, Ancona, Dahlberg and Wu independently succeeded in proving the BHP for a Lipschitz domain in late 1970's. Jerison-Kenig in 1982 extended the BHP to an NTA domain, whose boundary may be fractal. All of these works are rooted in Carleson's study on the local Fatou theorem in 1962, where the uniform barriers are ingeniously employed.

Surprisingly, Bass-Burdzy in 1991 proved the BHP for *Dirichlet irregular domains* such as a Hölder domain and a John domain. Their *box argument* requires no barriers. It is based on Brownian motion whose transition density is regarded as the Dirichlet heat kernel. If the heat kernel is bounded above and below by the product of the eigenfunctions with positive multiplicative constants depending on time, then the semigroup associated with the Dirichlet heat kernel is said to be *intrinsic ultracontractive (IU).* The IU can be considered to be a *parabolic boundary Harnack principle.* The validity of the IU also depends on the geometry of the domain. However, much worse domains can enjoy the IU. In fact, the IU holds for every bounded domain given by a graph, no matter how bad the graph is.

In this talk, we study the BHP and IU in parallel. We introduce *capaitary width*, which is closely related to the first eigenvalue of the Dirichlet Laplacian. First, we give unified Dini type conditions in terms of capaitary width for the BHP and IU. Second, we give more precise conditions under a suitable geometrical assumption on the domain. If the domain is described by the quasihyperbolic metric, then the conditions for the BHP and IU are very similar. If the domain is given by a graph, the conditions for the BHP and IU look very different. Sharpness of the conditions is demonstrated by explicit examples.

Interpolation of modified tent spaces and applications to elliptic boundary value problems

Alex Amenta, *Australian National University and Université Paris-Sud 11*

We consider a class of function spaces $T^{p,2}_s(X)$ associated to a metric measure space X. These spaces are a modification of the more classical tent spaces $T^{p,2}(X)$; the new parameter s controls an additional weight term, which behaves as a regularity parameter.

We identify the real and complex interpolation spaces between $T^{p_0,2}_{s_0}(X)$ and $T^{p_1,2}_{s_1}(X)$. There are no surprises with complex interpolation. However, real interpolation yields function spaces $L(p, \theta, 2)$ which are not tent spaces, but which appear in the study of elliptic BVPs with boundary data in Besov spaces (at least when $X = \mathbb{R}^n$). In particular, these spaces appear in the work of Barton and Mayboroda (2013).

We combine these identifications with recent work of Auscher and Stahlhut (2014) to derive certain *a priori* estimates for solutions to elliptic systems with complex measurable coefficients on half-spaces, with boundary data in Besov and Triebel–Lizorkin spaces.

H-distributions in various settings

Nenad Antonić, *University of Zagreb, Croatia*

H-distributions were recently introduced [4] as an extension of H-measures to the $\mathrm{L}^p - \mathrm{L}^q$ setting. As classical Tartar's H-measures (and similar objects independently introduced by Gérard under the name of microlocal defect measures), they are microlocal defect functionals defined on the tensor product of test functions in the physical variable $\mathbf{x} \in \mathbf{R}^d$ and the dual variable $\boldsymbol{\xi} \in \mathrm{S}^{d-1}$.

The original construction [4] can be extended to mixed-norm Lebesgue spaces [2], with a potential for further extension to Sobolev spaces modelled either over classical or mixed-norm Lebesgue spaces, or in the anisotropic direction as it was done in [3] for H-measures, depending on what might be required by intended applications.

For all above variants a localisation principle can be established, providing applications in the theory of partial differential equations like compactness by compensation [5] for equations with variable coefficients.

This is a joint work with Marko Erceg, Ivan Ivec, Marin Mišur and Darko Mitrović.

[1] Nenad Antonić, Marko Erceg, Marin Mišur: *On H-distributions*, in preparation.

[2] Nenad Antonić, Ivan Ivec: *On the Hörmander-Mihlin theorem for mixed-norm Lebesgue spaces*, submitted, 23 pp.

[3] Nenad Antonić, Martin Lazar: *Parabolic H-measures*, *Journal of Functional Analysis* **265** (2013) 1190–1239.

[4] Nenad Antonić, Darko Mitrović: *H-distributions - an extension of H-measures to an* $\mathrm{L}^p - \mathrm{L}^q$ *setting*, *Abstract and Applied Analysis* **2011** (2011) Article ID 901084, 12 pages.

[5] Marin Mišur, Darko Mitrović: *On a generalization of compensated compactness in the* $\mathrm{L}^p - \mathrm{L}^q$ *setting*, *Journal of Functional Analysis* **268** (2015) 1904–1927.

Variational parabolic capacity

Benny Avelin, *Aalto University, Finland and Uppsala University, Sweden*

In this talk I will present some recent results regarding nonlinear parabolic capacity related to the degenerate p-parabolic equation

$$\nabla \cdot (|\nabla u|^{p-2} \nabla u) = 0, \quad p > 2.$$

Specifically we introduce a variational quantity comparable to the nonlinear parabolic capacity introduced by Kinnunen, Korte, Kuusi and Parviainen -13 Math. Ann. An interesting byproduct of our proof is a local intrinsic version of our variational quantity allowing us to estimate the capacity of certain simple objects, for example cylinders, curves and certain surfaces.

Spectra of three-dimensional cruciform and lattice quantum waveguides

Fedor Bakharev, *Saint Petersburg State University, Russia*

We will establish that the discrete spectrum of Dirichlet Laplacian in the union of two orthogonal circular cylinders has the total multiplicity one. Also we are going to discuss the same problem for other shapes of cross-section. In particular it is proved that the multiplicity of the discreet spectrum depends on the shape of cross-section, and on the value of the angle between cylinders as well. In addition we prove that the homogeneous problem at the threshold of the spectrum has no bounded solutions. This information provides to give a one-dimensional model of a square lattice of thin quantum waveguides and to describe the asymptotic behaviour of spectral segments and gaps.

The work is performed as a part of the project 0.38.237.2014 SPSU. The author was supported by Chebyshev Laboratory (Department of Mathematics and Mechanics, St. Petersburg State University) under RF Government grant 11.G34.31.0026

N-term approximation of $BV_p^k([0,1)^d)$ functions by splines

Yuri Brudnyi, *Technion, Haifa, Israel*

The next problem goes back to the 1967 pioneering paper by Birman and Solomyak [BS].

Let $q = (\frac{k}{d} - \frac{1}{p})^{-1} < \infty$ be the limiting embedding exponent for the Sobolev space $W_p^k([0,1)^d)$. Do there exist a partition π_N of $[0,1)^d$ into N dyadic cubes and a piecewise polynomial $g_N(f)$ of degree $k-1$ on π_N such that

$$\|f - g_N(f)\|_q \leq cN^{-\frac{k}{d}} \max_{|\alpha|=k} \|D^\alpha f\|_p . \tag{1}$$

The answer is NO as (1) implies compactness of the embedding $W_p^k \subset L^q$ but becomes positive after replacing cubes in π_N by differences of dyadic cubes.

This result is firstly proved for the space $W_1^1([0,1)^2)$ and more general case of the space $BV([0,1)^2)$ in the 1999 breakthrowing paper [CDPY].

Following the line of the cited papers are proven the general approximation result for BV_p^k spaces generalizing the classical V_p spaces to the case of L^p functions and differences of higher order. As consequences we obtain analogous to (1) results for Sobolev spaces over BV and L^p, Besov and Triebel-Lizorkin spaces, and some results on the real interpolation of these spaces.

[BS] M. Birman and M. Solomyak, *Piecewise polynomial approximation of functions of classes* W_p^α, Math. USSR-Sb. 2(1967), 295-317.

[CDPY] A. Cohen et al., *Nonlinear approximation and the space* $BV(R^2)$, Amer. J. Math., 121 (1999), 587-628

Elliptic equations with measurable nonlinearity in nonsmooth domains

Sun-Sig Byun, *Seoul National University, Korea*

We consider a divergence form elliptic equation with measurable nonlinearity in a bounded non-smooth domain to study a global $W^{1,p}$ estimate for the weak solution. A minimal regularity requirement on the nonlinearity and a lower level of geometric assumption on the boundary are addressed for such a Calderón-Zygmund type estimate.

Weighted Lorentz spaces associated to a vector measure and interpolation with a parameter function

Ricardo del Campo Acosta, *Universidad de Sevilla, Spain*

It is well-known that if the classical Lions-Peetre real interpolation method $(\cdot,\cdot)_{\theta,q}$ $(0 < \theta < 1 \leq, q \leq \infty)$ is applied to a pair $(X, L^{\infty}(\mu))$, the result is the Lorentz space $L^{p,q}(\mu)$ with $p = \frac{1}{1-\theta}$, for every quasi-Banach space X such that $L^1(\mu) \subseteq X \subseteq L^{1,\infty}(\mu)$ and any scalar positive measure μ. See e.g. [2].

The aim of this work is to extend this result in a twofold direction: From scalar measures μ to vector measures m, and from the classical real interpolation method $(\cdot,\cdot)_{\theta,q}$ to general real interpolation methods $(\cdot,\cdot)_{\rho,q}$ associated to parameter functions ρ. For parameter functions ρ in certain classes of functions, these spaces $(X_0, X_1)_{\rho,q}$ were studied, first by Kalugina [4] and Gustavsson [3], and later by Persson [5], and other authors.

This extension procedure carries the necessity of introducing suitable Lorentz spaces $\Lambda_v^q(\|m\|)$ associated to a vector measure m and a weight v, which fit with our interpolation spaces. As a consequence of our interpolation results, we will find conditions under which such spaces are really normable quasi-Banach spaces.

Our approach is based on the relationship of the pair (ρ, q) with the *Ariño-Muckenhoupt weights* (see [1] and [6]), and sheds light even to the scalar measure case, providing a different point of view for it.

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Weighted estimates at the end-point

María J. Carro, *University of Barcelona, Spain*

The purpose of this talk is to present a method that allows us to obtain the boundedness of an operator

$$T : L^1 \longrightarrow L^{1,\infty}$$

from the boundedness of

$$T : L^2(w) \longrightarrow L^{2,\infty}(w),$$

as soon as we have this information for a sufficiently large class of weights w.

This class was defined in 1972 by B. Muckenhooupt and it is known as the A_2 class. The above method is an extension of the Rubio de Francia extrapolation theorem and it is joint work with L. Grafakos and J. Soria.

B. Muckenhoupt, *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc. 165 (1972), 207–226.

J. L. Rubio de Francia, *Factorization theory and A_p weights*, Amer. J. Math. 106 (1984), no. 3, 533–547.

Global Calderón–Zygmund type estimates for generalized p-Laplacian type elliptic equations

Yumi Cho, *Korea Institute for Advanced Study*

In this talk, we consider generalized p-Laplacian type equations

$$\begin{cases} \operatorname{div}\left(\frac{g(|Du|)}{|Du|}Du\right) & = & \operatorname{div}\left(\frac{g(|F|)}{|F|}F\right) & \text{in} & \Omega, \\ u & = & 0 & \text{on} & \partial\Omega. \end{cases}$$

Here, $g(0) = 0$ and $g : (0,\infty) \to (0,\infty)$ is of class $C^1(0,\infty)$ satisfying

$$0 < i_g \leq s_g < \infty,$$

where $i_g := \inf\limits_{t>0} \dfrac{tg'(t)}{g(t)}$ and $s_g := \sup\limits_{t>0} \dfrac{tg'(t)}{g(t)}$. We establish global Calderón-Zygmund type estimates under sufficient flatness of the boundary $\partial\Omega$ in the Reifenberg sense.

Logarithmic interpolation methods and Besov spaces

Fernando Cobos, *Universidad Complutense de Madrid, Spain*

In order to obtain Lorentz-Zygmund spaces $L_{p,q}(\log L)_b$ from the couple of Lebesgue spaces (L_{p_0}, L_{p_1}) one needs to use logarithmic perturbations of the real interpolation method $(A_0, A_1)_{\theta,q,\mathbb{A}}$, normed by

$$\|a\|_{(A_0,A_1)_{\theta,q,\mathbb{A}}} = \Big(\int_0^\infty [t^{-\theta} \ell^{\mathbb{A}}(t) K(t,a)]^q \frac{dt}{t} \Big)^{1/q}.$$

Here $1 \leq q \leq \infty, \mathbb{A} = (\alpha_0, \alpha_\infty) \in \mathbb{R}^2, \ell(t) = 1 + |\log t|, \ell^{\mathbb{A}}(t) = \ell^{\alpha_0}(t)$ if $0 < t \leq 1, \ell^{\mathbb{A}}(t) = \ell^{\alpha_\infty}(t)$ if $1 < t < \infty$ and now not only $0 < \theta < 1$ but also θ can take the values 0 and 1.

For $0 < \theta < 1$ the theory of spaces $(A_0, A_1)_{\theta,q,\mathbb{A}}$ is well-known, but if $\theta = 0$ or 1 some natural questions as the description of these spaces by means of the J-functional or duality have not been described yet. We do it in this talk and we also show some applications of logarithmic methods to problems on Besov spaces of logarithmic smoothness.

The talk is based on results of joint papers with A. Segurado [2] and O. Domínguez [1].

[1] F. Cobos and O. Domínguez, *On Besov spaces of logaritmic smoothness and Lipschitz spaces*, J. Math. Anal. Appl. 425 (2015) 71-84.

[2] F. Cobos and A. Segurado, *Description of logarithmic interpolation spaces by means of the J-functional and applications*, J. Funct. Analysis 268 (2015) 2906-2945.

A few more partial answers to a 51 year old interpolation question

Michael Cwikel, *Technion, Haifa, Israel*

It is now nearly 52 years since Studia Mathematica received Alberto Calderón's very remarkable paper about his theory of complex interpolation spaces. And one of the questions which Calderón implicitly asked in that paper, by solving it in a significant special case, is apparently still open today: Does complex interpolation preserve the compactness of an operator?

After briefly surveying attempts to solve this question over several decades, I will also report on a few new partial answers obtained during the past year, some of them (arXiv:1411.0171) jointly with Richard Rochberg. Among other things there is an interplay with Jaak Peetre's "plus-minus" interpolation method, (arXiv:1502.00986) a method which probably deserves to be better known.

Several distinguished mathematicians have expressed the belief that that the general answer to this question will ultimately turn out to be negative. Among other things, I will try to hint at where a counterexample might perhaps be hiding. You are all warmly invited to seek it out, or prove that it does not exist.

A fairly recent survey of this question is available at arXiv:1410.4527.

Bounded approximation by polynomials and the Rudin–Carleson theorem

Arthur Danielyan, *University of South Florida, Tampa, USA*

Let E be an arbitrary closed subset on the unit circle T of the complex plane $\mathbb{C}$ and let f be a continuous complex valued function on E. The problem of uniform approximation of f on E by polynomials $P_n(z)$ which are uniformly bounded on T has been considered by Zalcman in 1982. In this talk the complete solution to this problem will be presented. Our main result directly implies the classical Rudin-Carleson interpolation theorem. Further problems and applications will be discussed as well.

Reiteration of approximation spaces

Oscar Domínguez, *Universidad Complutense de Madrid, Spain*

Given a quasi-Banach space X and an approximation family $(G_n)_{n\in\mathbb{N}_0}$ of subsets of X, approximation spaces X_p^α are defined by selecting those elements of X such that $(n^{\alpha-1/p}E_n(f))$ belongs to ℓ_p. Here $\alpha > 0, 0 < p \leq \infty$ and $E_n(f)$ is the error of best approximation to f by the elements of G_{n-1}. These spaces have been studied by Butzer and Scherer [3], Brudnyi [1], Brudnyi and Kruglyak [2] and Pietsch [9] among other authors. Limiting approximation spaces $X_q^{(0,\gamma)}$ are defined by doing $\alpha = 0$ and inserting the weight $(1 + \log n)^\gamma$. They have been investigated by Cobos and Resina [6], Cobos and Milman [5] and Fehér and Grässler [8]. As it was shown in [6], even when $\gamma = 0$, the theory of limiting approximation spaces does not follow from the theory of spaces X_p^α by taking $\alpha = 0$. Spaces X_p^α and $X_q^{(0,\gamma)}$ allow to establish in an elegant and clear way a number of important results on function spaces, sequence spaces and spaces of operators.

In this talk we describe some recent results with F. Cobos [4] on reiteration of approximation constructions. The construction $(\cdot)_p^\alpha$ is stable by iteration [9] and a similar property holds for $(\cdot)_q^{(0,\gamma)}$ [8]. We study the stability properties when we apply first the construction $(\cdot)_p^\alpha$ and then $(\cdot)_q^{(0,\gamma)}$ or vice versa. As we will show, outside the case where $p = q$, the constructions do not commute. As application, we improve results by DeVore, Riemenschneider and Sharpley [7] about the relationship between smoothness of derivatives of f and the smoothness of f.

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One-scale H-measures, variants and applications

Marko Erceg, *University of Zagreb, Croatia*

Microlocal defect functionals (H-measures, H-distributions, semiclassical measures etc.) are objects which determine, in some sense, the lack of strong compactness for weakly convergent L^p sequences. In contrast to the semiclassical measures, H-measures are not suitable to treat problems with a characteristic length (e.g. thickness of a plate). Luc Tartar in his recent book overcame the mentioned restriction by introducing one-scale H-measures, a generalisation of H-measures with a characteristic length [2]. Moreover, these objects are also an extension of semiclassical measures, being functionals on continuous functions on a compactification of $\mathbb{R}^d \setminus \{0\}$.

We improve and generalise Tartar's localisation principle for one-scale H-measures from which we are able to derive the known localisation principles for both H-measures and semiclassical measures. Moreover, we develop a variant of compactness by compensation suitable for equations with a characteristic length [1].

Since one-scale H-measures are adequate only for the L^2 framework, we introduce the generalisation, one-scale H-distributions, as a counterpart of H-distributions with a characteristic length, and address some important features.

This is a joint work with Nenad Antonić and Martin Lazar.

[1] Nenad Antonić, Marko Erceg, Martin Lazar: *Localisation principle for one-scale H-measures,* submitted 35 pp.

[2] Luc Tartar: *Multi-scale H-measures, Discrete and Continuous Dynamical Systems, S* **8** (2015) 77–90.

The Dirichlet problem on bounded domains in a metric space with prime end boundary data

Dewey Estep, *University of Cincinnati, USA*

First introduced in the complex plane by Caratheodory, prime ends provide a way to define the boundary of a bounded domain such that its closure retains many properties intrinsic to the structure of the domain itself rather than its ambient space. For example, the prime end closure of the slit disk in **C** retains the structure imposed by the 'slit,' while the normal metric closure ignores it. Using the definition given by Adamowicz, Bjorn, Bjorn and Shanmugalingam, we may speak of prime ends in more general metric spaces. Here we define and study the Dirichlet problem with prime end boundary data on bounded domains, showing that under certain assumptions we may construct solutions using the Perron method.

Multiple and nodal solutions for nonlinear equations

Michael Filippakis, *University of Piraeus, Greece*

In this paper we consider a nonlinear parametric Dirichlet problem driven by a nonhomogeneous differential operator (special cases are the p-Laplacian and the (p, q)-differential operator) and with a reaction which has the combined effects of concave ($(p-1)$-sublinear) and convex ($(p-1)$-superlinear) terms. We do not employ the usual in such cases AR-condition. Using variational methods based on critical point theory, together with truncation and comparison techniques and Morse theory (critical groups), we show that for all small $\lambda > 0$ (λ is a parameter), the problem has at least five nontrivial smooth solutions (two positive, two negative and the fifth nodal). We also prove two auxiliary results of independent interest. The first is a strong comparison principle and the second relates Sobolev and Hőlder local minimizers for C^1 functionals.

Acknowledgement. The publication of this paper has been partly supported by the University of Piraeus Research Center.

A two dimensional asymptotic model of the wall of a blood vessel

Arpan Ghosh, *Linköping University, Sweden*

We derive a two dimensional model of the wall of a blood vessel having a circular cross section of a fixed radius all along its length while having a general curvature and torsion. The wall consists of three elastic, anisotropic layers, called intima, media and adventia, which except intima are formed of bundles of collagen fibres. We first choose a suitable orthonormal moving frame of reference in order to have relatively simpler expressions. Constitutive relations of elasticity and Newton's second law provide us with a PDE system for the displacement of the wall material. A dynamic boundary condition and a kinematic no-slip boundary condition are assumed on the inner surface of the wall which help in coupling the equations with the Navier-Stokes equation governing the blood flow within the vessel. We assume the thickness of the wall of the vessel to be very small compared to the radius and the length of the vessel. Under such an assumption, we use asymptotic expansions to obtain a model which essentially describes the wall of the vessel as a two dimensional surface.

A necessary and sufficient condition for the continuity of local minima of parabolic variational integrals with linear growth

Ugo Gianazza, *University of Pavia, Italy*

For proper weak solutions u of the parabolic 1-laplacian equation

$$u \in C_{\rm loc}\big(0,T;L^2_{\rm loc}(E)\big) \cap L^1_{\rm loc}\big(0,T;W^{1,1}_{\rm loc}(E)\big)$$
$$u_t - \operatorname{div}\Big(\frac{Du}{|Du|}\Big) = 0 \qquad \text{locally in } \ E_T,$$

we establish a necessary and sufficient condition for u to be continuous at a point, in terms of a sufficient fast decay of the local integral of the gradient Du. These equations arise also as minima of parabolic variational integrals with linear growth with respect to $|Du|$. Hence, the continuity condition continues to hold for such minima. This is a joint work with Emmanuele DiBenedetto and Colin Klaus (Vanderbilt University, Nashville, USA)

A sense preserving homeomorphism of a cube may have a.e. negative Jacobian

Paweł Goldstein, *University of Warsaw and Polish Academy of Science*

The importance of the class of a.e. approximately differentiable homeomorphisms is justified by the result of Federer, who proved that such homeomorphisms, if they additionally possess Lusin's N property, constitute valid variable changes under a Lebesgue integral. The problem with such homeomorphisms (and in general with weakly differentiable homeomorphisms) is that, in contrast to the case with diffeomorphisms, it is unclear if (and how) the topological properties (being a homeomorphism, for a start, or being sense preserving) relate to the analytic ones (e.g. the sign of the Jacobian determinant). These questions are important e.g. in nonlinear elasticity.

In a joint result with Piotr Hajłasz, we construct an a.e. approximately differentiable homeomorphism H of an n-dimensional unit cube onto itself, that

(a) is sense-preserving,
(b) is measure preserving (thus it has Lusin's N property),
(c) both H and H^{-1} are uniform limits of measure and sense preserving diffeomorphisms,
(d) on a subset of full measure the approximate derivative of H is equal to the fixed reflection R; $R(x_1, \ldots, x_n) = (x_1, \ldots, x_{n-1}, -x_n)$.

We give yet another example, that satisfies (a), (b) and (c), coincides with a fixed reflection on a subset of positive measure and is bi-Hölder continuous with an arbitrary exponent.

New regularity results for elliptic PDEs and application to adaptive approximation methods

Markus Hansen, *Technical University Munich, Germany*

We present some new results on the regularity of solutions to elliptic PDEs in Lipschitz domains with polyhedral structure. More precisely, these solutions belong to certain weighted Sobolev spaces, which in turn are embedded into Besov or Triebel-Lizorkin spaces. Both scales are known to be closely related to Approximation spaces w.r.t. adaptive approximation methods (n-term wavelet approximation and adaptive Finite element methods).

These regularity results are subsequently combined with approximation results for these adaptive methods to derive convergence rates for adaptive methods for elliptic PDEs. As a second application, the previous results are utilized for the approximation of parametric problems and elliptic problems with random coefficients.

Truncated Riesz potential estimates for elliptic equations with drift terms

Takanobu Hara, *Tokyo Metropolitan University, Japan*

We consider divergence form elliptic equations with a strongly singular drift term -$div(A\nabla u) + \mathbf{b}\cdot\nabla u = f$ in a domain $\Omega \subset \mathbb{R}^n$ ($n \geq 3$). We give a weak-type $L^1 - L^{n/(n-2),\infty}$ estimate for a solution to the Dirichlet problem with the homogeneous boundary condition. Moreover, we give a two-sided pointwise potential estimate.

Extension and trace problems for Besov and Triebel–Lizorkin functions

Lizaveta Ihnatsyeva, *University of Jyväskylä, Finland*

Recently there have been introduced several analogues for Besov spaces and Triebel–Lizorkin spaces in a quite general setting, which, in particular, includes certain topological manifolds, fractals, graphs and Carnot-Carathéodory spaces. Employing one of the available definitions, we study extension domains for Besov-type and for Triebel-Lizorkin type functions in the setting of a metric measure space with a doubling measure; as a special case we obtain a characterization of extension domains for classical Besov spaces defined via L^p-modulus of smoothness. The talk is based on joint work with Toni Heikkinen and Heli Tuominen.

Convolutions and approximations

Daniyal Israfilov, *Balikesir University, Turkey*

In the space $L_{2\pi}^{p(\cdot)}$ with $p(\cdot) \in \beta_{2\pi}$, where by $\beta_{2\pi}$ we denote the class of the exponents $p(\cdot)$, for which

$$|p(x) - p(y)| \leq \frac{c_0}{-\log(|x-y|)}, \quad x, y \in [0, 2\pi] \quad \text{with} \quad |x-y| \leq 1/2,$$

we define a mean value operator σ_h

$$(\sigma_h f)(x,u) := \frac{1}{2h} \int_{-h}^{h} f(x+tu)\, dt, \ 0 < h < \pi, \ x \in [0, \pi], \ -\infty < u < \infty,$$

which is linear and bounded in $L_{2\pi}^{p(\cdot)}$.

For $f \in L_{2\pi}^{p(\cdot)}$ we define the best approximation number

$$E_n(f)_{p(\cdot)} := \inf_{T_n} \|f - T_n\|_{p(\cdot)}$$

by trigonometric polynomials of degree at most n and a convolution type operator

$$\int_{-\infty}^{\infty} (\sigma_h f)(\cdot, u)\, d\sigma(u)$$

with a bounded variation function $\sigma(u)$ on the real line $\mathbb{R}$. Denoting

$$D(f, \sigma, h, p(\cdot)) := \left\| \int_{-\infty}^{\infty} (\sigma_h f)(\cdot, u)\, d\sigma(u) \right\|_{p(\cdot)},$$

we estimate the quantity $D(f, \sigma, h, p(\cdot))$ using the best approximation number $E_n(f)_{p(\cdot)}$.

In the variable exponent Lebesgue spaces a convolution is defined and its estmations in the variable exponent Lebesgue spaces by the best approximation numbers are obtained.

Theorem *If* $f \in L_{2\pi}^{p(\cdot)}$ *with* $p(\cdot) \in \hat{\text{ß}}_{2\pi}$, *then*

$$D(f, \sigma, h, p(\cdot)) \leq c \sum_{k=0}^{m} E_{2^k-1}(f)_{p(\cdot)}\, \delta_{2^k,h} + c_{p(\cdot)} E_{2^{m+1}}(f)_{p(\cdot)}$$

for every $m \in \mathbb{N}$, *where*

$$\begin{aligned} \delta_{2^k,h} \ &:= \sum_{l=2^k}^{2^{k+1}-1} |\hat{\sigma}(lh) - \hat{\sigma}((l+1)h)| + \left|\hat{\sigma}\left(2^k h\right)\right|, \\ \hat{\sigma}(x) \ &:= \int_{-\infty}^{\infty} \frac{\sin(ux)}{ux} d\sigma(u), \quad 0 < h < \pi. \end{aligned}$$

This is joint work with my graduate student Elife Yirtici.

Extension theorems dealing with weighted Orlicz-Slobodetskii space

Agnieszka Kałamajska, *Polish Academy of Sciences and University of Warsaw*

Having given weight $\rho = \tau(\mathrm{dist}(x, \partial\Omega))$ defined on Lipschitz boundary domain Ω and Orlicz function R, we construct the weight $\omega_\rho(\cdot,\cdot)$ defined on $\partial\Omega \times \partial\Omega$ and extension operator Ext from certain subspace of weighted Orlicz–Slobodetski space $Y^{R,R}_{\omega_\rho}(\partial\Omega)$ subordinated to the weight ω_ρ to Orlicz–Sobolev space $W^{1,R}_\rho(\Omega)$. The weight $\omega_\rho(\cdot,\cdot)$ is independent of R. This gives the new tool to deal with boundary value problems like:

$$\begin{cases} -\mathrm{div}\,(\rho(x)B(\nabla u(x))) = f & \text{in} \quad \Omega \\ u = g & \text{in} \quad \partial\Omega. \end{cases} \tag{2}$$

with inhomogeneous boundary data provided in the weighted Orlicz setting. Result is new in the unweighted Orlicz setting for general function R as well as in the weighted L^p setting.

Asymptotically sharp Bernstein type inequalities for polynomials and rational functions on different sets of the complex plane

Sergei Kalmykov, *University of Szeged, Hungary*

Bernstein (or Riesz) type polynomial inequalities are well known. On the complex plane Bernstein inequality was extended to compact sets bounded by smooth Jordan curves and the asymptotically sharp constant can be expressed with the normal derivative of Green's function [1]. There is a general conjecture for Jordan arcs that the asymptotically sharp Bernstein factor can be expressed as the maximum of the two normal derivatives of Green's function. It was proved for the subarcs [2], and later, general subsets of the unit circle [3]. In this talk we begin with the case of polynomials on arbitrary analytic Jordan arcs and then continue with more general cases of rational functions on C^2 smooth Jordan arcs and curves. The proofs of the main results extensively use facts from potential and interpolation theories, Borwein-Erdélyi inequality for derivative of rational functions on the unit circle, Gonchar-Grigorjan estimate of the norm of holomorphic part of meromorphic functions, and Totik's construction of fast decreasing polynomials (see [4]).

This is based on a joint work with Béla Nagy.

The work was supported by the European Research Council Advanced grant No. 267055.

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Finite dimensional Chebyshev subspaces of spaces of discontinuous functions

Aref Kamal, *Sultan Qaboos University, Oman*

In this paper the author studies the existence and the characterization of the n dimensional Chebyshev subspaces of $L^\infty[a,b], B[a,b]$, and some other spaces of discontinuous functions. In the case when the space admits an n dimensional Chebyshev subspaces, the author develops a complete characterization for those n dimensional Chebyshev subspaces. In the case when the space does not admit an n dimensional Chebyshev subspaces, the author proves it.

Commutators and algebras of convolution type operators on weighted Lebesgue spaces

Yuri Karlovich, *Universidad Autónoma del Estado de Morelos, Mexico*

Let $\mathfrak{B}_{p,w}$ denote the Banach algebra of all bounded linear operators acting on the weighted Lebesgue space $L^p(\mathbb{R}, w)$ where $1 < p < \infty$ and w is a Muckenhoupt weight. We study the Banach subalgebra $\mathfrak{A}_{p,w}$ of $\mathfrak{B}_{p,w}$ generated by all multiplication operators aI and all convolution operators $W^0(b) = \mathcal{F}^{-1}b\mathcal{F}$, where $\mathcal{F}$ is the Fourier transform, the functions $a, b \in L^\infty(\mathbb{R})$ admit piecewise slowly oscillating discontinuities on $\mathbb{R} \cup \{\infty\}$ and b is a Fourier multiplier on $L^p(\mathbb{R}, w)$. First, applying results on commutators of pseudodifferential operators with non-regular symbols, we study the compactness of commutators $[aI, W^0(b)]$ for several subclasses of functions a, b. Then, applying the Allan-Douglas local principle, the theory of Mellin pseudodifferential operators with symbols of limited smoothness, a symbol calculus for Banach algebras generated by two idempotents, quantitative characteristics of Muckenhoupt weights and techniques of limit operators, we construct a noncommutative Fredholm symbol calculus for Banach algebra $\mathfrak{A}_{p,w}$ and obtain a Fredholm criterion for the operators $A \in \mathfrak{A}_{p,w}$ in terms of their Fredholm symbols.

Planar Sobolev approximation and extension

Pekka Koskela, *University of Jyväskylä, Finland*

A full geometric characterization for a bounded simply connected planar domain to be a $W^{1,p}$-extension domain, $p = 2$, was established some 25 years ago. The case of $p > 2$ was completed about five years ago. We describe the solution for the remaining case $1 < p < 2$, and hopefully also the endpoint $p = 1$. For $p = 1$, new density results are necessary.

Conditions for the absence of point eigenvalues in the axisymmetric problem of a freely floating moonpool

Nikolay Kuznetsov, *Russian Academy of Sciences, St. Petersburg, Russia*

We analyse the spectral problem describing the coupled time-harmonic motion of the following mechanical system: an axisymmetric moonpool (toroidal, surface-piercing body) floats freely in infinitely deep water bounded above by a free surface. The system's motion is of small amplitude near equilibrium which allows us to apply the linearized model developed by John [1] and substantially simplified in [3]. The essential point of the problem under consideration is that one of the coupling conditions is linear with respect to the spectral parameter (the frequency of oscillations), whereas two other problem's relations (another coupling condition and the free surface boundary condition) are quadratic with respect to it.

It is shown in [4], that every positive number is a point eigenvalue when a motionless (but floating freely) body belongs to a certain family depending on the eigenvalue. The family is constructed by virtue of the so-called semi-inverse procedure and a characteristic feature of bodies belonging to it is that their axisymmetric immersed parts divide the free surface into at least two connected components and are bulbous on both sides (directed to infinity and to the inside).

Our present aim is to describe frequency intervals containing no point eigenvalues provided a freely floating moonpool satisfies a geometric condition that excludes the bulbous geometry described above. This condition was used in [2] for guaranteeing the absence of point eigenvalues in the problem when the same moonpool is fixed instead of floating freely. The frequency intervals obtained here only partly coincide with those in [2] because an extra condition is involved — the equation of body's motion.

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[3] N. Kuznetsov & O. Motygin, 2011 *J. Fluid Mech.* **679**, 616–627.
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One-dimensional interpolation inequalities, Carlson–Landau inequalities and magnetic Schrödinger operators

Ari Laptev, *Institut Mittag-Leffler, Stockholm and Imperial College, London*

We shall prove refined first-order interpolation inequalities for periodic functions and give applications to various refinements of the Carlson–Landau-type inequalities and to magnetic Schrödinger operators. We also obtain Lieb-Thirring inequalities for magnetic Schrödinger operators on multidimensional cylinders.

Hardy–Sobolev inequalities on general open sets

Juha Lehrbäck, *University of Jyväskylä, Finland*

The aim of this talk is to present both sufficient and necessary conditions for the validity of the so-called Hardy–Sobolev inequality

$$\left(\int_G |f|^q \, \delta_{\partial G}^{(q/p)(n-p+\beta)-n} \, dx\right)^{1/q} \leq C \left(\int_G |\nabla f|^p \, \delta_{\partial G}^{\beta} \, dx\right)^{1/p}$$

for all $f \in C_0^\infty(G)$, where $G \subset \mathbb{R}^n$ is an open set and $\delta_{\partial G}(x)$ denotes the distance from $x \in G$ to the boundary ∂G. For $1 \leq p < n$ and $p \leq q \leq p^* = np/(n-p)$ (and weight exponent $\beta \in \mathbb{R}$) these inequalities form a natural interpolating scale between the (weighted) Sobolev inequality, which is the case $q = p^*$, and the (weighted) Hardy inequality, which is the case $q = p$. The Assouad dimension of the complement G^c turns out to play an important role in both sufficient and necessary conditions, and the sufficient conditions are natural generalizations of the previously known inequalities where G^c is an m-dimensional subspace of $\mathbb{R}^n$. This talk is based on a joint work with Antti Vähäkangas; see arxiv.org/abs/1502.01190 for a related preprint.

Quasi-linear PDEs and low-dimensional sets

John Lewis, *University of Kentucky, Lexington, USA*

In this talk we discuss recent results concerning boundary Harnack inequalities and the Martin boundary problem, for non-negative solutions to equations of p-Laplace type with variable coefficients, vanishing on the boundary of certain lower dimensional sets. We establish our quantitative and scale-invariant estimates in the context of m dimensional Reifenberg flat sets, $1 \leq m \leq n-2$. In this setting such sets are removable for p Laplace type operators when $n - p \geq m$, as follow for example from the theory of p capacities in Adams - Hedberg. Thus we consider only $p > n - m$. Our work generalizes earlier work of Lewis, Nyström, and Lundström in the more traditional setting of $n-1$ dimensional Reifenberg sets.

On theorems of F. and M. Riesz

Elijah Liflyand, *Bar-Ilan University, Ramat-Gan, Israel*

We discuss various analogs of the famous theorem due to F. and M. Riesz on the absolute continuity of the measure whose negative Fourier coefficients are all zeros. A simpler and more direct proof of one of such analogs is obtained. In the same spirit a different proof is found for another theorem of F. and M. Riesz on absolute continuity. These results are closely related to one theorem of Hardy and Littlewood on the absolute convergence of the Fourier series of a function of bounded variation whose conjugate is also of bounded variation and its extensions to the non-periodic case. Certain multidimensional results are discussed as well.

On p-harmonious functions with variable radius

José G. Llorente, *Universitat Autònoma de Barcelona, Spain*

In the last years some efforts have been made to clarify the stochastic framework associated to some remarkable nonlinear differential operators, as the p-laplacian or the ∞-laplacian, by means of appropriate(nonlinear) mean value properties. It turns out that the mean value property given by the following convex combination

$$u(x) = \frac{\alpha}{2}\big(\sup_{B(x,r)} u + \inf_{B(x,r)} u\big) + \frac{(1-\alpha)}{m(B(x,r))}\int_{B(x,r)} u\,dm \qquad (*)$$

(here $0 \leq \alpha \leq 1$ and m stands for Lebesgue measure) is closely related to the p-laplacian ($0 \leq \alpha = \alpha(p) < 1$) or the ∞-laplacian ($\alpha = 1$). Let $\Omega \subset \mathbb{R}^n$ be a bounded domain and suppose that for each $x \in \Omega$ we are given a radius $r = r(x)$ such that $B(x,r) \subset \Omega$. Solutions of $(*)$ have sometimes been called p-harmonious functions in Ω. The authors study conditions on the domain Ω and on the radius function $r(x)$ under which the Dirichlet problem with continous boundary data has a unique, continous p-harmonious solution in Ω. This provides an intrinsic generalization of previous results of Manfredi-Parviainen-Rossi and Luiro-Parviainen-Saksman when $r(x) \equiv const.$, in the sense that the radii are variable and no extension of the domain is required. (Joint work with Angel Arroyo, Universitat Autònoma de Barcelona).

On the Benjamin and Lighthill conjecture for steady water waves with vorticity

Evgeniy Lokharu, *Linköping University, Sweden*

In 1954, Benjamin and Lighthill made a conjecture concerning irrotational steady gravity waves on water of finite depth. According to this conjecture, all steady water waves with zero vorticity may be parametrized by points in some cusped region on the (r, s)-plane (r and s are the non-dimensional Bernoullis constant and the flow force, respectively) and any point of the region corresponds to some steady wave motion. We prove this conjecture for near-critical steady waves in the presence vorticity.

This is a joint work with Vladimir Kozlov (Linköping University) and Nikolay Kuznetsov (Russian Academy of Sciences).

Perron's method for the porous medium equation

Teemu Lukkari, *University of Jyväskylä, Finland*

O. Perron introduced his celebrated method for the Dirichlet problem for harmonic functions in 1923. The method produces two solution candidates for given boundary values, an upper solution and a lower solution. A central issue is then to determine when the two solutions are actually the same function. The classical result in this direction is *Wiener's resolutivity theorem:* the upper and lower solutions coincide for all continuous boundary values. We discuss the resolutivity theorem and the related notions for the porous medium equation

$$u_t - \Delta u^m = 0$$

in the degenerate case $m > 1$ in general cylindrical domains. This is joint work with J. Kinnunen (Aalto) and P. Lindqvist (NTNU Trondheim).

A Phragmén-Lindelöf theorem for p-subharmonic functions

Niklas L. P. Lundström, *Umeå University, Sweden*

Suppose that $\Lambda \subset \mathbf{R}^n$ is an m-dimensional hyperplane with $0 \leq m \leq n-1$ and let $p \in (n-m, \infty]$. Suppose that u is p-subharmonic in $\mathbf{R}^n \setminus \Lambda$ with $\limsup_{x \to \Lambda} u(x) \leq 0$. Then, we prove that either $u \leq 0$ in $\mathbf{R}^n \setminus \Lambda$ or it holds that

$$\liminf_{R \to \infty} \frac{\sup_{|x|=R} u(x)}{R^\beta} > 0$$

where $\beta = (p-n+m)/(p-1)$ with $\beta = 1$ if $p = \infty$. The above growth rate is sharp. Moreover, we give some related results for p-harmonic functions.

Functions with low rank Hessians

Jan Malý, *Charles University in Prague and Jan Evangelista Purkyně University in Ústí nad Labem, Czech Republic*

We present a new method how to construct examples of Sobolev functions with low rank Hessians. We show a strictly convex example or a low rank Hessian function which approximates a given function. These examples complement some recent results by Robert Jerrard and Reza Pakzad on the Monge-Ampère equation and developable mappings. This is a joint work with Zhuomin Liu and Reza Pakzad.

Sobolev-type embeddings in metric measure spaces via generalized Poincaré inequalities

Lukáš Malý, *University of Jyväskylä, Finland*

We will discuss self-improvement properties of (generalized) Poincaré inequalities based on rearrangement-invariant Banach function spaces over doubling metric measure spaces and the consequent Sobolev-type embeddings. The talk is based on joint work with Toni Heikkinen and the presented theory generalizes results of Cianchi, Hajłasz–Koskela, MacManus–Pérez, as well as others.

Given an r.i. space $X = X(\mathcal{P}, \mu)$ and an open set $\Omega \subset \mathcal{P}$, we define a fractional John–Nirenberg functional by

$$\|u\|_{A^s_X(\Omega)} = \sup_{\mathbf{B}} \left\| \sum_{B \in \mathbf{B}} \frac{\chi_B}{\mu(B)^{1/s}} \fint_B |u(x) - u_B| \, d\mu(x) \right\|_{X(\Omega)},$$

where the supremum is taken over all (at most) countable collections of pairwise disjoint balls in Ω. We will find the optimal Marcinkiewicz space Z such that $\|u - u_B\|_{Z(B)} \lesssim \|u\|_{A^s_X(\lambda B)}$. Alternatively, we will determine the optimal modulus of continuity ω such that $|u(x) - u(y)| \lesssim \omega(\mu(B))\|u\|_{A^s_X(\lambda B)}$ holds for a.e. $x, y \in B$. These inequalities will then be applied to prove that

$$\|u - u_B\|_{Y(B)} \lesssim \|g\|_{X(\lambda B)} \quad \text{or} \quad |u(x) - u(y)| \lesssim \omega(\mu(B))\|g\|_{X(\lambda B)}$$

for suitable r.i. spaces X and Y, provided that the couple of functions (u, g) satisfies a certain Poincaré inequality, while the value of the parameter s is specified by the upper regularity dimension of the doubling measure.

In particular, in metric measure spaces that support a 1-Poincaré inequality, the Sobolev-type embedding $\|u\|_{Y(B)} \lesssim \|g\|_{X(B)}$ follows from the boundedness of a Hardy-type operator $H_s : \overline{X}(0, \mu(B)) \to \overline{Y}(0, \mu(B))$, which links our results to the optimal Sobolev embeddings in $\mathbb{R}^s$ by Edmunds–Kerman–Pick.

BV functions on curves in metric measure spaces

Olli Martio, *Finnish Academy of Science and Letters, Helsinki*

A new modulus, AM–modulus, for a curve family Γ is introduced in a metric measure space (X, d, ν). It is shown that a function of bounded variation (BV) in the sense of M. Miranda Jr. is of bounded variation on AM almost every curve in X. Properties of the AM–modulus are studied. There is also a Banach space $BV_{AM}(X)$ of BV functions $u \in L^1(X)$ on curves in X whose functions are not defined as limits of locally Lipschitz or Newtonian functions in X. Only minimal assumptions on the metric measure space (X, d, ν) are used.

Interpolation formulas for the Fremlin tensor product

Mieczysław Mastyło, *Adam Mickiewicz University and Polish Academy of Sciences* (*Poznań branch*)

We will discuss interpolation of the Fremlin tensor product as well as spaces of regular multilinear forms and operators. We show applications to factorization of matrices with respect to the Schur product. Our results imply various abstract variants of Schur's classical result. In particular we extend Pisier's converse for matrices in finite dimensional ℓ_p-spaces to the setting of complex Calderón interpolation of finite dimensional Banach lattices. The talk is based on joint work with Andreas Defant.

Calderón–Zygmund theory on a class of uniformly rectifiable subdomains of Riemannian manifolds

Marius Mitrea, *University of Missouri, Columbia, USA*

In this talk I shall explore how the geometry of a given ambient shapes up the type of analytic results one can expect in that environment. Specifically, the main thesis is that appropriate uniform rectifiability and infinitesimal flatness conditions imposed for the boundary of an open set imply well-posedness results for a large variety of elliptic boundary value problems formulated with data in L^p spaces for arbitrary $p \in (1, \infty)$. In many ways, such a result is in the nature of best possible. In the process, we develop a brand of Calderón-Zygmund theory on Riemannian manifolds which is effective in dealing with the very rough settings described above.

A semilinear elliptic problem with a singularity at $u = 0$

François Murat, *Université Pierre et Marie Curie (Paris VI) and CNRS*

In this joint work with Daniela Giachetti (Rome) and Pedro J. Martinez Aparicio (Cartagena, Spain) we consider the elliptic semilinear equation with homogeneous Dirichlet boundary condition

$$-divA(x)Du = F(x,u) \text{ in } \Omega,$$

$$u = 0 \text{ on } \partial\Omega,$$

where the nonlinearity $F(x,u)$ is singular at $u = 0$, with a singularity of the type

$$F(x,u) = \frac{f(x)}{u^\gamma} + g(x)$$

with $\gamma > 0$ and f and g non negative (which implies that also the solution u is non negative).

The main difficulty is to give a convenient definition of the solution of this problem, in particular when $\gamma > 1$.

We give such a definition and we prove the existence and stability of a solution, as well as its uniqueness when $F(x,u)$ is non increasing in u.

We also consider the homogenization problem where Ω is replaced by Ω^ε, with Ω^ε obtained by removing from Ω many very small holes in such a way that passing to the limit when ε tends to zero, the homogeneous Dirichlet boundary condition on $\partial\Omega^\varepsilon$ leads to an homogenized problem where a "strange term" μu appears in Ω.

This work has been inspired by the paper of Lucio Boccardo and Luigi Orsina *Semilinear elliptic equations with singular nonlinearities*, Calculus of Variations and Partial Differential Equations, 37, (2010), 363–380, and by the paper of Lucio Boccardo and Juan Casado-Diaz *Some properties of solutions of some semilinear elliptic singular problems and applications to the G-convergence*, Asymptotic Analysis, 86, (2104), 1–15.

Comparison of Navier and Dirichlet fractional Laplacians

Alexander I. Nazarov, *Russian Academy of Sciences and St. Petersburg University*

Let Ω be a bounded domain with smooth boundary. We compare two natural types of fractional Laplacians $(-\Delta)^s$, namely, the "Navier" and the "Dirichlet" ones. We denote their quadratic forms by $Q^N_{s,\Omega}$ and $Q^D_{s,\Omega}$, respectively.

Theorem 1. Let $s > -1$, $s \notin \mathbb{N}_0$. Then for $u \in \mathrm{Dom}(Q^D_{s,\Omega})$, $u \not\equiv 0$, the following relations hold:

$$Q^N_{s,\Omega}[u] > Q^D_{s,\Omega}[u], \quad \text{if} \quad 2k < s < 2k+1, \quad k \in \mathbb{N}_0;$$
$$Q^N_{s,\Omega}[u] < Q^D_{s,\Omega}[u], \quad \text{if} \quad 2k-1 < s < 2k, \quad k \in \mathbb{N}_0.$$

Moreover, for $u \in \mathrm{Dom}(Q^D_{s,\Omega})$ the following facts hold (here $F(\Omega)$ stands for the class of smooth and bounded domains containing Ω).

$$Q^D_{s,\Omega}[u] = \inf_{\Omega' \in F(\Omega)} Q^N_{s,\Omega'}[u], \quad \text{if} \quad 2k < s < 2k+1, \quad k \in \mathbb{N}_0;$$
$$Q^D_{s,\Omega}[u] = \sup_{\Omega' \in F(\Omega)} Q^N_{s,\Omega'}[u], \quad \text{if} \quad 2k-1 < s < 2k, \quad k \in \mathbb{N}_0.$$

Theorem 2. Let $0 < |s| < 1$, and let $f \in \mathrm{Dom}(Q^D_{s,\Omega})$, $f \geq 0$, $f \not\equiv 0$. Then the following relations hold:

$$(-\Delta_\Omega)^s_N f > (-\Delta_\Omega)^s_D f, \quad \text{if} \quad 0 < s < 1;$$
$$(-\Delta_\Omega)^s_N f < (-\Delta_\Omega)^s_D f, \quad \text{if} \quad -1 < s < 0.$$

Here all inequalities are understood in the sense of distributions.

This talk is based on joint papers with Roberta Musina, see [1], [2]. Author was supported by RFBR grant 14-01-00534 and by St.Petersburg University grant 6.38.670.2013.

[1] R. Musina, A.I. Nazarov, On fractional Laplacians // Comm. in PDEs, **39** (9) (2014) 1780–1790.

[2] R. Musina, A.I. Nazarov, On fractional Laplacians–2, 2014. Preprint arXiv:1408.3568.

Restrictions of functions and formal power series in n variables to lines in $\mathbb{C}^n$

Tejinder Neelon, *California State University San Marcos, USA*

I will present generalizations and analogs of the following classical theorems in functions spaces and subrings of formal power series.

Siciak's theorem. *A C^∞ function on $\mathbb{R}^n$ that is real analytic on every line is real analytic.*

Zorn-Lelong's theorem. *If a double power series $F(x, y)$ over $\mathbb{C}$ converges on a set of lines of positive capacity then $F(x, y)$ is convergent.*

Abhyankar-Moh-Sathaye's theorem. *The transfinite diameter of the convergence set of a divergent power series, with coefficients in an arbitrary valued field, is zero.*

The convergence set of a divergent formal power series $f(x_1, \ldots, x_n)$ is defined to be the set of all "directions" $\xi \in \mathbb{P}^{n-1}$ along which f is absolutely convergent. We prove that if a set $\Lambda \subseteq \mathbb{P}^{n-1}$ is of positive logarithmic capacity in $\mathbb{C}^n$, then any function f whose restriction along every direction $\xi \in \Lambda$ belongs, in a *uniform way*, to a fixed ultradifferentiabe function class $C_{[\mathcal{M}]}(\mathbb{R})$, necessarily belongs to $C_{[\mathcal{M}]}(\mathbb{R}^n)$. The formal power series analog is also obtained. In particular, it's shown that the logarithmic capacity of the convergence set of a divergent power series $F(x_1, \ldots, x_n)$ is always zero.

My joint work with Daowei Ma considers the collection $\mathrm{Conv}(\mathbb{P}^{n-1})$ of convergence sets of divergent power series in n variables. We show that $\mathrm{Conv}(\mathbb{P}^{n-1})$ contains projective hulls of compact pluripolar sets and countable unions of algebraic varieties in $\mathbb{P}^{n-1}$. Every convergence set (of a divergent power series) is a countable union of projective hulls of compact pluripolar sets and every countable union of closed complete pluripolar sets in $\mathbb{P}^{n-1}$ is a convergence set.

Gradient and potential estimates near Dini-flat boundary points

Kaj Nyström, *Uppsala University, Sweden*

Consider a bounded domain $\Omega \subset \mathbb{R}^n$ and $x_0 \in \partial\Omega$. We establish gradient and potential estimates, at x_0, for solutions to general quasilinear equations of p-Laplace type. We prove, under restrictions on the data ψ, that Dv is non-tangentially continuous at x_0, whenever $\partial\Omega$ is Dini-flat at $x_0 \in \partial\Omega$ and v is a solution to

$$\begin{cases} \operatorname{div} a(Dv) = 0 & \text{in } \Omega \\ \qquad v = \psi & \text{on } \partial\Omega\,. \end{cases}$$

Under similar assumptions we prove that x_0 is a Lebesgue point of Du, and that $Du(x_0)$ is bounded, when u solves general measure data problems. Quantitative estimates in terms of Wolff and Riesz potentials are derived. Our results are derived using only Dini-type flatness assumptions, for the domain and its boundary, at the point x_0. Our results seem to be new already in the case $p = 2$ and for the Laplace equation.

A semi discrete Schrödinger–Poisson equation in $\mathbb{R}^3$

Maria-Eleni Poulou, *University of Aegean, Ermoupolis, Greece*

We consider a semi-discrete in time relaxation scheme to discretize a damped forced Schrödinger-Poisson equation. This provides us with a discrete infinite-dimensional dynamical system for which we prove the existence of a finite dimensional global attractor.

Logistic age-structured population model in a changing environment

Sonja Radosavljevic, *Linköping University, Sweden*

We present an age-structured logistic population model derived from the classical McKendrick-von Foerster model by including intra-species competition for resources or overcrowding effects. These effects are incorporated in the model through a logistic term in the balance equation. Under assumptions that the vital rates and the carrying capacity are age and time-dependent, we prove that the model has a unique nonnegative bounded solution. Moreover, we investigate asymptotic behavior of the number of newborns and of the total population in a changing environment. Our results show that if the net reproductive rate is larger than one, the total number of newborns and the total population tend to certain positive constant values. Otherwise, both functions tend to zero. In addition, we investigate asymptotic behavior of a solution under the assumption that the vital rates are periodically changing and that the carrying capacity is a constant.

This is a joint work with Uno Wennergren, IFM, Linköping University and Vladimir Kozlov, MAI, Linköping University.

Finite rank operators in the Fock space and the $\bar{\partial}$ equation in certain spaces of distributions

Grigori Rozenblioum, *Chalmers University of Technology, Göteborg, Sweden*

The important theorem by Hörmander declares that if a function f on $\mathbb{C}$ grows not too fast at infinity then the equation $\bar{\partial}g = f$ admits a solution also with controlled growth at infinity. In the dual problem, when the given function f decays sufficiently fast at infinity and the solution g is saught also in a class of decaying functions, is solvable only under the additional condition that f is orthogonal to all analytic polynomials. We consider a more general problem on the solvability of the $\bar{\partial}$ equation in some classes of distributions, continuous functionals on functions with controlled growth. The theorem on solvability is applied to the problem on characterization of finite rank Toeplitz operators with distributional symbol in the Fock space.

Cyclicity and optimal polynomials

Daniel Seco, *University of Warwick, UK*

For functions f in Dirichlet-type spaces over the disk we study how to determine constructively the optimal polynomials p_n of degree n, in terms of the norm of $p_n f - 1$, concentrating on the case when f is a cyclic function. We then give upper and lower bounds for the ratio of convergence of this norm to zero as n approaches ∞. Then we will introduce a few new results about similar spaces over the bidisk.

Compactness results for limiting interpolation methods

Alba Segurado, *Universidad Complutense de Madrid, Spain*

Interpolation theory plays an important role in the study of function spaces, operator theory and approximation theory, among many other fields of mathematics. Many of these applications are based on the real method $(A_0, A_1)_{\theta,q}$ introduced by Lions and Peetre (see [1]), where $0 < \theta < 1$. Limiting methods where θ can take the values 0 or 1 and A_0 is continuously embedded in A_1 (that is, in the ordered case) are also very useful when decomposing spaces by means of simpler ones and in the study of the boundedness of operators between spaces with complicated structures and of singular integrals (see [7, 2]).

To be in the ordered case is essential for the techniques used in those papers, but, from the point of view of interpolation theory, it is only a restriction. For this reason, it is natural to study the extension of limiting methods to arbitrary, not necessarily ordered, couples of Banach spaces $\bar{A} = (A_0, A_1)$. In [4, 5] the present author and Fernando Cobos suggested an extension of these methods for arbitrary couples that allows one to produce a sufficiently rich theory, and also studied some of its properties.

On the other hand, interpolation of compact operators is a classical question that has attracted the attention of many authors. As concerns the real method, the final result was obtained by Cwickel [6] and Cobos, Kühn and Schonbek [3], who proved that if $T \in \mathcal{L}(\bar{A}, \bar{B})$ and any of its restrictions $T : A_j \to B_j$ $(j = 0, 1)$ is compact, then the interpolated operator $T : (A_0, A_1)_{\theta,q} \to (B_0, B_1)_{\theta,q}$ is also compact. However, things work differently when dealing with limiting methods.

In this talk, we will review the definitions of the mentioned methods, show some examples and study the behaviour of compact operators under these methods. The results are part of a joint work with Fernando Cobos ([4]).

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Haar series and Paley spaces

Evgeniy Semenov, *Voronezh State University, Russia*

A Banach space E of measurable functions on $[0,1]$ is said to be rearrangement invariant (r.i. in short), or symmetric, if

(i) $|x(t)| \leqslant |y(t)|$ for all $t \in [0,1]$ and $y \in E$ imply $x \in E$ and $\|x\|_E \leqslant \|y\|_E$;

(ii) the equimeasurability of x and y and $y \in E$ imply $x \in E$ and $\|x\|_E = \|y\|_E$.

As usual, we shall assume that E is separable or E is a dual to a separable space. We also assume that the normalization condition $\|\varkappa_{[0,1]}\|_E = 1$ holds, where $\varkappa_e$ is the characteristic function of a measurable set $e \subset [0,1]$. In any r.i. space E the dilation operator

$$\sigma_\tau x(t) = \{\ x(t/\tau),\ 0 \leqslant t \leqslant \min(1,\tau), 0 \ \text{ for other } t \in [0,1] \quad , \tau > 0$$

boundedly acts and the numbers

$$\alpha_E = \lim_{\tau \to 0} \frac{\ln \|\sigma_\tau\|_E}{\ln \tau}, \quad \beta_E = \lim_{\tau \to \infty} \frac{\|\sigma_\tau\|_E}{\ln \tau}$$

are called the Boyd indices of E. We always have $0 \leqslant \alpha_E \leqslant \beta_E \leqslant 1$. For example, $\alpha_{L_p} = \beta_{L_p} = 1/p$ for all $p \in [1,\infty]$. The Orlicz space generated by the function $M(u) = e^{u^2} - 1$ is denoted by $\exp L_2$, we denote the closure of L_∞ in $\exp L_2$ by G. The functions $\chi_0^0(t) = 1$,

$$\chi_n^k(t) = \begin{cases} 2^{n/2}, & (k-1)2^{-n} < t < (k-1/2)2^{-n} \\ -2^{n/2}, & (k-1/2)2^{-n} < t < k2^{-n} \\ 0 & \text{for other } t \in [0,1] \end{cases}$$

forms the Haar system (H.s. in short), where $1 \leqslant k \leqslant 2^n$, $n = 0,1,2,\ldots$. The formula $m = 2^n + k$ establishes a one-to-one correspondence between $\{(0,0),(k,n), 1 \leqslant k \leqslant 2^n, n = 0,1,2\ldots\}$ and the set of integers $\mathbb{N}$. It allows to use the one-index system $\{\chi_m, m \in \mathbb{N}\}$.

The H.s. forms a complete orthonormal system which is a monotone basis in any separable r.i. space. The H.s. is an unconditional basis in a separable r.i. space E iff $0 < \alpha_E \leqslant \beta_E < 1$. In this case $\|x\|_E$ and $\|Px\|_E$ are equivalent. Here

$$x(t) = \sum_{m=1}^{\infty} c_m \chi_m(t), \quad Px(t) = \left(\sum_{m=1}^{\infty} (c_m \chi_m(t))^2 \right)^{1/2}.$$

Given an r.i. space E, by $P(E)$ we denote the Paley space endowed with the norm

$$\|x\|_{P(E)} = \|Px\|_E.$$

Denote

$$Rad = \{x : x = \sum_{k=1}^{\infty} a_k r_k(t), \sum_{k=1}^{\infty} a_k^2 < \infty\},$$

where $r_k(t) = \operatorname{sign} \sin 2^k \pi t$, $k \in \mathbb{N}$ are the Rademacher functions. Clearly,

$$\left\| \sum a_k r_k \right\|_{P(E)} = \|a\|_{l_2}.$$

We present the main properties of the Paley spaces. Let E be an r.i. space.

1. $P(E)$ is an r.i. space iff $E = L_2$.
2. $P(E)$ is an r.i. space up to equivalence iff $0 < \alpha_E \leqslant \beta_E < 1$.
3. *Rad* is a 1–complemented subspace of E.
4. The inclusion $P(E) \subset E$ holds iff $\alpha_E > 0$.
5. The inclusion $P(E) \supset E$ holds iff $0 < \alpha_E \leqslant \beta_E < 1$.

6. $P(E)$ is reflexive iff E is reflexive.

We study relations between the spaces $P(L_\infty)$, G, BMO and diadic BMO.

Joint work with S. A. Astashkin.

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Taut strings and real interpolation

Eric Setterqvist, *Linköping University, Sweden*

If we try to approximate a function from L^p by the ball of L^∞, we discover that the element of best approximation is invariant with respect to the L^p-norm, $1 \le p \le \infty$. This simple fact lies at the core of the Marcinkiewicz interpolation theorem. In this talk we will present different classes of domains with such property. It turns out that these domains are closely connected to the classical Hardy-Littlewood-Pólya majorization and K-monotonicity of the couple (L^1, L^∞).

The notion of taut string was introduced by G.B. Dantzig in 1971 in connection with problems in optimal control. Later on, taut strings have found important applications in different areas of applied mathematics, especially in statistics and image processing. We will show that taut strings and their continuous and multidimensional generalizations are closely related to domains that have invariant element of best approximation with respect to the L^p-norm.

The talk is based on joint work with Natan Kruglyak.

Sphericalization and flattening transformations of metric measure spaces

Nageswari Shanmugalingam, *University of Cincinnati, USA*

In this talk we will discuss two dual transformations on metric measure spaces that convert a bounded space into an unbounded space and vice versa, and discuss the analytic and geometric properties that are preserved by these transformations. This talk is based on joint work with Xiangdong Xie, David Herron, and Xining Li.

A characterization of the Gaussian Lipschitz space

Peter Sjögren, *University of Gothenburg, Sweden*

In the n-dimensional Ornstein-Uhlenbeck setting, a Lipschitz space was defined by Gatto and Urbina, in terms of the gradient of the Ornstein-Uhlenbeck Poisson integral of the function. We show that this space can also be described as a Lipschitz space in the ordinary sense, by means of an inequality for the modulus of continuity. The proof is based on several sharp estimates for the Ornstein-Uhlenbeck Poisson kernel and its gradient, also of independent interest.

This is joint work with Liguang Liu (Beijing).

On the non-vanishing property for real analytic solutions of the p-Laplace equation

Vladimir G. Tkachev, *Linköping University, Sweden*

The main result of the present talk is motivated by the non-vanishing property for real analytic solutions to the p-Laplace equation and was inspired by the following question of John Lewis [4]: Does there exist a real homogeneous polynomial $u(x)$ of degree $m = \deg u \geq 2$ in $\mathbb{R}^n$, $n \geq 3$ satisfying

$$\Delta_p u := |Du|^2 \Delta u + \frac{p-2}{2} \langle Du, D|Du|^2 \rangle = 0, \tag{3}$$

where $p > 1$, $p \neq 2$? Lewis itself answered in negative this question in two dimensions in [4]. On the other hand, notice that for any $d \geq 2$ and $n \geq 2$ there exist plenty quasi-polynomial $C^{d,\alpha}$-smooth solutions of (3) in $\mathbb{R}^n$ [3], [1], [2], [8], [6].

We have the following particular answer on the Lewis question.

Theorem 1. *Any real homogeneous cubic polynomial solution of (3) in $\mathbb{R}^n$ for $n \geq 2$ is identically zero.*

The proof of Theorem 1 makes use of a nonassociative algebra argument developed earlier for similar problems in [7], [5].

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Hölder continuity of quasiminimizers with nonstandard growth

Olli Toivanen, *Polish Academy of Sciences, Warsaw, Poland*

I will talk on a recent manuscript with Tomasz Adamowicz, available as arXiv:1503.02599, where we show the Hölder continuity of quasiminimizers of the energy functional $\int_\Omega f(x,u,\nabla u)\,dx$ with nonstandard growth under the general structure conditions

$$|z|^{p(x)} - b(x)|y|^{r(x)} - g(x) \leq f(x,y,z) \leq \mu|z|^{p(x)} + b(x)|y|^{r(x)} + g(x).$$

The result is illustrated by showing that weak solutions to a class of (A,B)-harmonic equations

$$-\mathrm{div}A(x,u,\nabla u) = B(x,u,\nabla u)$$

are quasiminimizers of the variational integral of the above type and, thus, are Hölder continuous. The results extend work by Chiadò Piat–Coscia, Fan–Zhao and Giusti–Giaquinta.

Estimates of the Neumann–Laplace operator first eigenvalue for conformal regular domains

Alexander Ukhlov, *Ben-Gurion University of the Negev, Israel*

We study eigenvalues of the Neumann-Laplace boundary value problem (the free membrane problem) in simply connected conformal regular domains $\Omega \subset \mathbb{C}$

$$-\Delta u = \lambda u \text{ in } \Omega,$$
$$\left.\frac{\partial u}{\partial n}\right|_{\partial\Omega} = 0.$$

A simply connected plane domain Ω is called a conformal α-regular domain if there exists a conformal mapping $\varphi : \Omega \to \mathbb{D}$ such that

$$\iint\limits_{\mathbb{D}} |(\varphi^{-1})'(w)|^\alpha \; dudv < \infty \text{ for some } \alpha > 2.$$

A plane domain is called conformal regular if it is conformal α-regular for some $\alpha > 2$.

Theorem A. *Let $\Omega \subset \mathbb{C}$ be a conformal α-regular domain. Then the spectrum of Neumann-Laplace problem in Ω is discrete, can be written in the form of a non-decreasing sequence*

$$0 < \lambda_1[\Omega] \leq \lambda_2[\Omega] \leq ... \leq \lambda_n[\Omega] \leq ... ,$$

and

$$1/\lambda_1[\Omega] \leq \frac{16}{\sqrt[\alpha]{\pi}} \left(\frac{2\alpha-2}{\alpha-2}\right)^{\frac{2\alpha-2}{\alpha}} \|\psi' \mid L^\alpha(\mathbb{D})\|^2 \tag{4}$$

where $\psi = \varphi^{-1} : \mathbb{D} \to \Omega$ is the Riemann conformal mapping of the unit disc $\mathbb{D} \subset \mathbb{C}$ onto Ω.

This study is based on universal weighted Poincaré-Sobolev inequalities with conformal weights. As a consequence we obtain the classical Poincare-Sobolev inequalities and the lower estimates of the first eigenvalues of the Neumann-Laplace operator in conformal regular domains.

This is joint work with Vladimir Gol'dshtein.

Quasi-diagonalization of Hankel operators

Dimitri Yafaev, *Université de Rennes I, France*

We show that all Hankel operators H realized as integral operators with kernels $h(t+s)$ in $L^2(\mathbb{R}_+)$ can be quasi-diagonalized as $H = \mathsf{L}^*\Sigma\mathsf{L}$. Here L is the Laplace transform, Σ is the operator of multiplication by a function (distribution) $\sigma(\lambda)$, $\lambda \in \mathbb{R}$. We find a scale of spaces of test functions where L acts as an isomorphism. Then L^* is an isomorphism of the corresponding spaces of distributions. We show that $h = \mathsf{L}^*\sigma$ which yields a one-to-one correspondence between kernels $h(t)$ and sigma-functions $\sigma(\lambda)$ of Hankel operators. The sigma-function of a self-adjoint Hankel operator H contains substantial information about its spectral properties. Thus we show that the operators H and Σ have the same numbers of positive and negatives eigenvalues. In particular, we find necessary and sufficient conditions for sign-definiteness of Hankel operators. These results are illustrated at examples of quasi-Carleman operators generalizing the classical Carleman operator with kernel $h(t) = t^{-1}$ in various directions. The concept of the sigma-function directly leads to a criterion (equivalent of course to the classical Nehari theorem) for boundedness of Hankel operators. Our construction also shows that every Hankel operator is unitarily equivalent by the Mellin transform to a pseudo-differential operator with amplitude which is a product of functions of one variable only (of $x \in \mathbb{R}$ and of its dual variable).