p-harmonic functions on metric spaces

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Abstract

Nonlinear potential theory of p-harmonic functions has been studied since about 1970. A p-harmonic function u is locally minimizing $\int |\nabla u|^p$ with $p > 1$. Much of the harmonic theory has been extended to p-harmonic functions in weighted $\mathbf{R}^n$ in the monograph by Heinonen, Kilpeläinen and Martio (1993).

In metric spaces the theory is even younger. Heinonen and Koskela (Acta Math., 1998) introduced upper gradients in metric spaces, which has lead Cheeger and Shanmugalingam to develop a generalization of Sobolev spaces to metric spaces and enabled the study of p-harmonic functions in metric spaces.

The theory of p-harmonic functions (and related things like p-superharmonic functions) in metric spaces has been developed by Aikawa, Heinonen, Kinnunen, Koskela, Latvala, MacManus, Martio, Rajala, Shanmugalingam, the speakers and probably some more that we have forgotten (we apologize). Some of the results obtained are new also in the Euclidean theory.

In this talk, we will give a survey of the theory of p-harmonic functions in metric spaces. A particular emphasize will be given on boundary regularity.