p-HARMONIC APPROXIMATION OF FUNCTIONS OF LEAST GRADIENT

PETRI JUUTINEN

Our goal is to establish a natural connection between the minimizers of two closely related variational problems. We prove global and local convergence results for the p-harmonic functions as $p \to 1$, and show that the limit function minimizes (at least locally) the total variation of the vector-valued measure ∇u in $BV(\Omega)$. Continuous functions with this property are usually called functions of least gradient.

The aforementioned results appear quite natural, especially after noticing that

$$\|\nabla u\|(\Omega) = \int_\Omega |\nabla u| \, dx$$

for $u \in W^{1,1}(\Omega)$. However, some caution is needed in the proofs, because functions of least gradient differ from p-harmonic functions in many aspects. Most importantly, the characterization of p-harmonic functions in terms of the p-Laplace equation shows that the property of being p-harmonic is completely local. The same is not true for the functions of least gradient since their superlevel sets must be area-minimizing, and that is not a local property.

Department of Mathematics and Statistics, P.O.Box 35, FIN-40014 University of Jyväskylä, Finland

E-mail address: peanju@maths.jyu.fi