Regularity of p-harmonic functions in Euclidean spaces and Heisenberg groups

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Abstract

It is well known that p-harmonic functions have Hölder continuous derivatives for $1 < p < \infty$. In this talk we first survey regularity results for p-harmonic functions in $\mathbb{R}^N$. We then consider the generalizations of the p-Dirichlet integral of the type

$$\int_\Omega \left(\Lambda^2 + |\mathfrak{X}u|^2\right)^{p/2} \, dx$$

where $\Lambda \geq 0$, $\Omega \subset \mathbb{R}^N$ is a given domain, and $\mathfrak{X}u = (X_1 u, X_2 u, \ldots, X_k u)$ is the gradient of u relative to a frame of linearly independent vector fields $\mathfrak{X} = \{X_1, X_2, \ldots, X_k\}$ in $\mathbb{R}^N$.

An important class of examples is given by Carnot groups, the simplest of which is the Heisenberg group $\mathcal{H}^n$. In this case $\mathfrak{X}$ is the horizontal frame consisting of $2n$ linearly independent left-invariant horizontal vector fields and $N = 2n+1$. Estimating the missing derivative is a serious obstacle when trying to extend the classical regularity results to this setting. This is largely an open question.

We will discuss in some detail the geometry and analysis of $\mathcal{H}^n$ and present two regularity results in the lower dimensional case $\mathcal{H}^1$: estimates of Cordes type that give Hölder continuous derivatives for p near 2 and a mixed Moser iteration scheme that works in the nondegenerate case $\Lambda > 0$ for the range $2 \leq p < 4$