

**Double covers of Klein surfaces with an automorphism group of type
(2, 2, 2, n)**

Abstract We say that a group G of automorphisms of a compact Klein surface X is of type $(2, 2, 2, n)$ if the universal covering transformation group of G is an NEC group with quadrangle signature $(2, 2, 2, n)$. Alternatively, the orbit space X/G is a hyperbolic quadrilateral with angles $\pi/2, \pi/2, \pi/2$ and π/n . Let X^+ be the Riemann double cover of X . It is well known that the full group $Aut(X^+)$ of conformal and anticonformal automorphisms of X^+ contains a subgroup iso-morphic to $Aut(X) \times C_2$, where $Aut(X)$ is the full group of automorphisms of X . Then a natural question arises: is $Aut(X^+)$ equal to $Aut(X) \times C_2$, or does X^+ admit other automorphisms?

This question has been considered for bordered Klein surfaces X of algebraic genus g with $12(g - 1)$ automorphisms by May, and with $8(g - 1)$ automorphisms by Bujalance, Costa, Gromadzki and Singerman. In both cases, $Aut(X)$ is of type $(2, 2, 2, n)$, with $n = 3$ in the first case and $n = 4$ in the second. Later, Costa and Porto studied the more general case where n is an odd prime p . They proved that $Aut(X^+) = Aut(X) \times C_2$ almost always occurs for such actions, and that in fact there is just one single exception for each value of p .

In this talk we will show that the same holds for group actions of type $(2, 2, 2, n)$ for all $n \geq 6$, again with one single exception for each value of n . This is joint work (in progress) with Emilio Bujalance and Marston Conder.