Topology Optimization in Solid and Fluid Mechanics

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ABSTRACT

Summary This talk concerns recent developments in topology optimization. Work in this area has almost exclusively been concerned with design of solids and structures. Here extensions to fluid mechanics, with possible applications in biomechanics, are given.

Optimization strategies are present everywhere in engineering and nature. When designing industrial products the designer tries out a sequence of designs in an attempt to gradually improve the product, i.e., even if not explicitly stated as such, an optimization is performed. In nature, evolution of biological systems follows paths that we can imagine are governed by an attempt to optimize some performance index. The mathematical paradigm that models optimization strategies in engineering as well as in nature is **Optimization with State Constraints (OSC)**.

The general form of an optimization problem with state constraints (a problem from OSC) is the following:

$$\min_{x,u} \quad f(x,u)$$
$$\text{subject to} \quad \begin{cases} \mathcal{S}(x,u) = 0 \\ g(x,u) \leq 0. \end{cases}$$

Here f is the performance measure which is a function of two types of variables: the design variable x and the state variable u. The design variable is what can change when the industrial product or biological system is modified, and for each such design, the system will take on a certain state determined by the state variable. For a given design the state is defined by a mathematical system which we abstractly write $\mathcal{S}(x,u) = 0$ and call the analysis or state problem. This system will frequently be a partial or ordinary differential equation with its origin in continuum mechanics and is what is traditionally solved in computational engineering. The novelty of OSC is to surround this problem with an optimization goal. Finally, $g(x,u) \leq 0$ can represent any type of explicit constraints on the variables.

This talk will give several explicit examples of the above structure as well as discuss numerical procedures and present solutions. The most direct example of a problem from OSC, much studied within structural optimization, is the case of a truss structure, where u represents a displacement vector and x is a vector of cross-section areas of bars. The state problem, $\mathcal{S}(x,u) = 0$, takes the form

$$K(x)u = F, \tag{1}$$

where F is the prescribed load vector and $K(x)$ is the stiffness matrix, which will depend linearly on x. A frequently used performance measure is $f(x,u) = \frac{1}{2}F^T u$, which we interpret as the flexibility or the negative of the stiffness of the structure. One also easily concludes that this performance measure equals minus the equilibrium potential energy of the structure. Furthermore, it has the property that structures optimized by its use have uniformly distributed stress, implying that available material is used efficiently. This type of structural optimization is usually called topology optimization since trusses can be excluded from the optimal design by letting $x \approx 0$ and in this way an optimal connection or topology is found.

The continuum analogy of the truss topology optimization problem is when $\mathcal{S}(x, u) = 0$ represents the equations of linear elasticity. The problem then becomes substantially more complicated. In fact, a naive extension results in a non-well posed problem which lacks a solution. To remedy this, different types of regularisations are possible, as extensively discussed by Borrvall [1]. Furthermore, how to obtain a stable finite element discretization is not obvious and certain similarities with mixed finite elements in, for instance, Stokes flow exist. In Figure 1 a large-scale continuum topology optimization problem is shown.

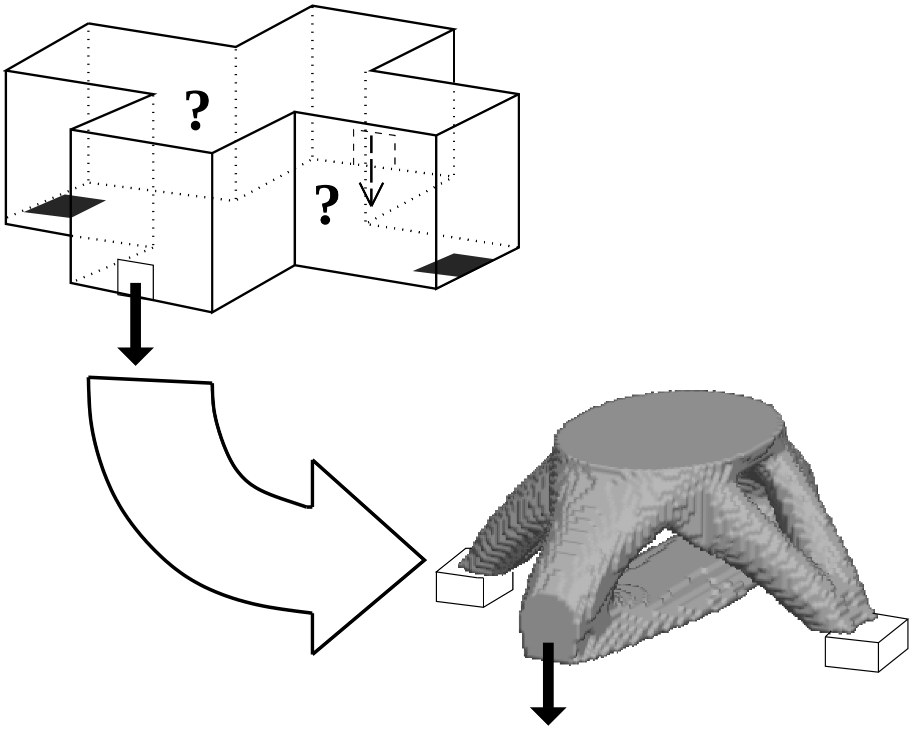

Figure 1: A continuum topology optimization problem. The cross-shaped domain should be partly filled by a prescribed amount of material to obtain an as stiff structure as possible.

A few years ago our attention was drawn to a series of papers (see, e.g. Karch et al. [3]) on modeling of arterial vascular trees, e.g. the corona artery in humans. Here a method based on optimality reasoning (but in our view, without formulating a clear overall goal in the sense of OSC) for constructing arterial trees is given. We almost immediately realized the possibility of transferring our knowledge in truss topology optimization to this domain, and subsequently published the paper [4]. Indeed, a very close analogy can be constructed: the stiffness objective becomes the objective of minimizing power losses for a prescribed flow; in the state problem, equation (1), u plays the role of pressure and F represents prescribed outflows of fluid. The stiffness matrix becomes a matrix representing fluid flow resistance, which if Hagen-Poiseuille flow is assumed will depend on the second power of cross-sectional areas, and, thus, not linearly as in the truss case. In Figure 2 an example of arterial tree optimization is shown.

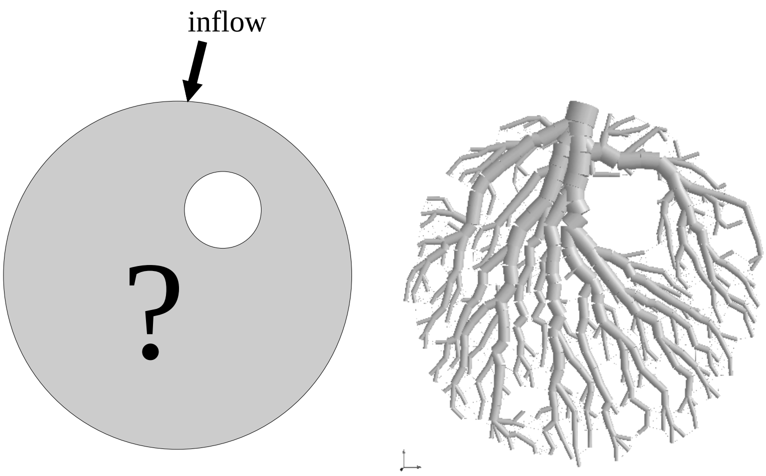

Figure 2: An optimal arterial tree problem: inflow and outflow is prescribed; the tree represents that of minimum pressure loss.

The next natural extension of this line of research is to do topology optimization in the fluid continuum case, see [2]. The goal is to determine at what places of a predetermined design domain there should be fluid or not (i.e. solid) in order to extremize a power objective and subject to a given amount of fluid. Possible applications include design for minimum head loss in pipe bends, diffusers and valves, optimal conceptual design of air flow channels in aerial vehicles, as well as design of submerged bridge pillars for minimum environmental impact on watercourse flows. The state equation is in computations so far taken to be Stokes system governing very viscous flow. An example is shown in Figure 3.

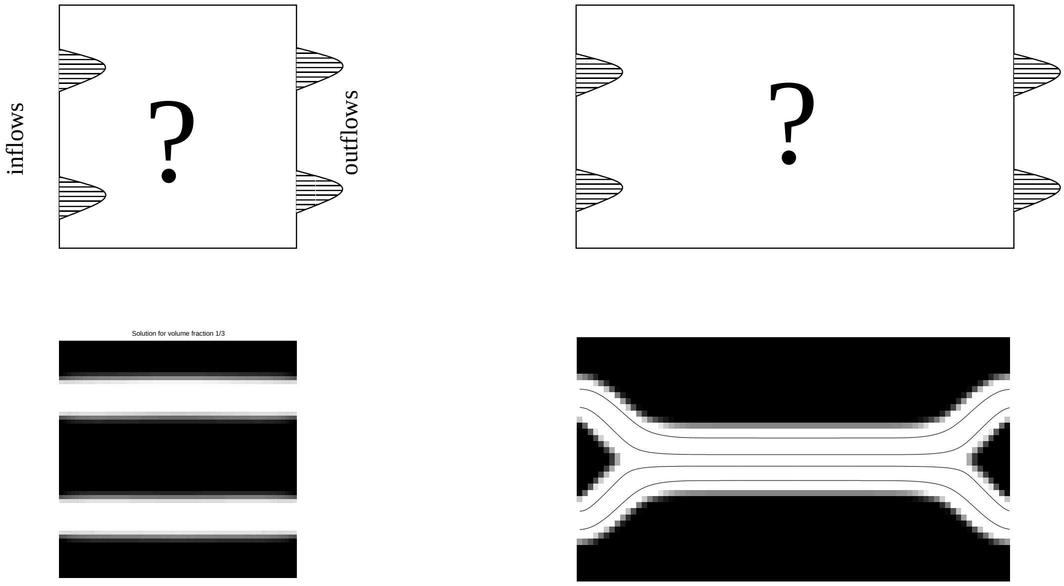

Figure 3: The system of channels giving minimum pressure loss for prescribed in- and out-flow is found. It is concluded that the length of the domain influences the topology.

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