Title: Quasi-Cauchy-Riemann equations

Abstract: A system of quasi-Cauchy-Riemann equations (qCR) is a system of first order PDE's of the form $(\det G)^{-1}G\nabla k = (\det\tilde{G})^{-1}\tilde{G}\nabla\tilde{k}$, where $G = \alpha qq^T + \beta q^T + q\beta^T + \gamma$ and $\tilde{G} = \tilde{\alpha}qq^T + \tilde{\beta}q^T + q\tilde{\beta}^T + \tilde{\gamma}$ are elliptic coordinates matrices and k and $\tilde{k}$ are dependent variables. These equations come up naturally as a characterization of a wide class of quasi-potential Newton equations $\ddot{q} = -(\det G)^{-1}G\nabla k = -(\det\tilde{G})^{-1}\tilde{G}\nabla\tilde{k}$, that admit n quadratic integrals of motion and appear to be a natural generalization of the classical separable potential systems when $\tilde{G} = Id$.
For $n = 2$ and a suitable choice of matrices G and $\tilde{G}$, the qCR equations become the well known Cauchy-Riemann (CR) equations. Like the CR equations these equations exhibit a remarkable algebraic structure: solutions can be multiplied in a similar way as solutions to the CR equations can be multiplied through multiplication of analytic functions.
I will present qCR, and describe the multiplicative structure on its solution space.